

# dB math

Training materials for wireless trainers



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This talk is about decibels and how to use them, compared to mW.

Duration: 30 minutes

Version 1.8 by Carlo, @2009-11-25

Version 1.11 by Rob, @2010-03-03

Version 1.12 by Rob, @2010-03-12

# Goals

- ▶ Electromagnetic waves carry power measured in milliwatts.
- ▶ Decibels (dB) use a relative logarithmic relationship to reduce multiplication to simple addition.
- ▶ You can simplify common radio calculations by using dBm instead of mW, and dB to represent variations of power.
- ▶ It is simpler to solve radio calculations in your head by using dB.

# Power

- ▶ Any electromagnetic wave carries energy - we can feel that when we enjoy (or suffer from) the warmth of the sun. The amount of energy received in a certain amount of time is called **power**.
- ▶ The electric field is measured in V/m (volts per meter), the power contained within it is proportional to the square of the electric field:

$$P \sim E^2$$

- ▶ The unit of power is the **watt (W)**. For wireless work, the **milliwatt (mW)** is usually a more convenient unit.

The power  $P$  is of key importance for making wireless links work: you need a certain minimum power in order for a receiver to make sense of the signal.

In practice, we measure the power by means of some form of receiver, e.g. an antenna and a voltmeter, power meter, spectrum analyzer, or even a radio card and laptop.

# Gain & Loss

- ▶ If the amplitude of an electromagnetic wave increases, its power increases. This increase in power is called a **gain**.
- ▶ If the amplitude decreases, its power decreases. This decrease in power is called a **loss**.
- ▶ When designing communication links, you try to maximize the gains while minimizing any losses.

You “gain” signal at the transmitter, using an amplifier, and when using a properly aligned antenna.

You “lose” signal at connectors, transmission lines, and naturally as waves propagate through the environment.

# Intro to dB

- ▶ Decibels are a **relative** measurement unit unlike the absolute measurement of milliwatts.
- ▶ The ***decibel (dB)*** is 10 times the decimal logarithm of the ratio between two values of a variable. The calculation of decibels uses a logarithm to allow very large or very small relations to be represented with a conveniently small number.
- ▶ On the logarithmic scale, the reference cannot be zero because the log of zero does not exist!

# Why do we use dB?

▶ Power does not fade in a linear manner, but inversely as the square of the distance.

▶ You move by  $x$  and the signal decreases by  $1/x^2$ ; hence, the **“inverse square law.”**

1 meter away → some amount of power

2 meters away → 1/4 power at one meter

4 meters away → 1/16 power at one meter

8 meters away → 1/64 power at one meter

▶ The fact that exponential relationships are involved in signal strength measurement is one reason why we use a logarithmic scale.

# Inverse square law

- ▶ The ***inverse square law*** is explained by simple geometry. The radiated energy expands as a function of the distance from the transmitter.

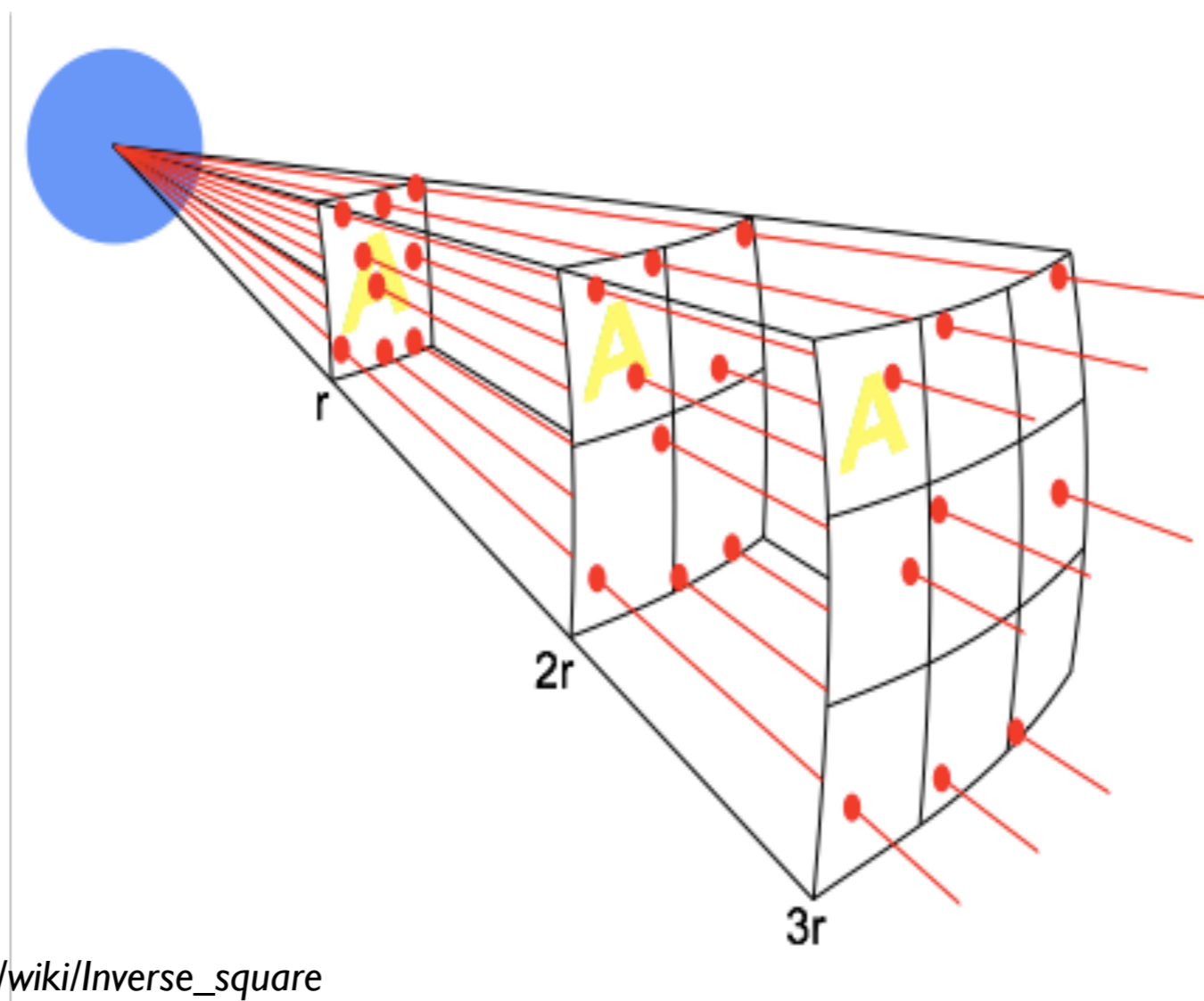


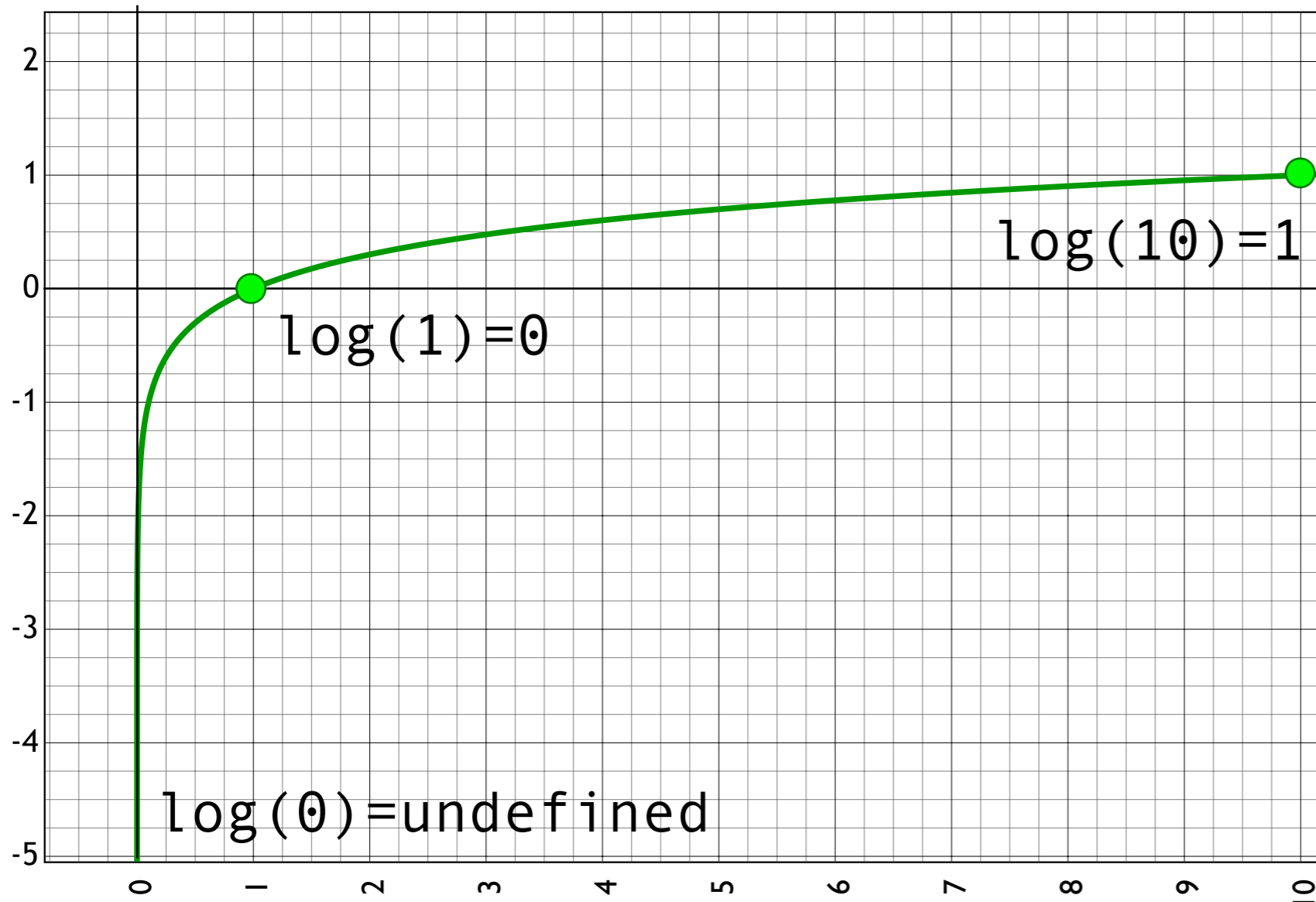
Figure from [http://en.wikipedia.org/wiki/Inverse\\_square](http://en.wikipedia.org/wiki/Inverse_square)

This law says that some physical quantity or strength is inversely proportional to the square of the distance from the source of that physical quantity.

The inverse-square law generally applies when some force, energy, or other conserved quantity is radiated outward radially from a source. Since the surface area of a sphere (which is  $4\pi r^2$ ) is proportional to the square of the radius, as the emitted radiation gets farther from the source, it must spread out over an area that is proportional to the square of the distance from the source. Hence, the radiation passing through any unit area is inversely proportional to the square of the distance from the source.

( from [http://en.wikipedia.org/wiki/Inverse\\_square](http://en.wikipedia.org/wiki/Inverse_square) )

# A quick review of logarithms



The **logarithm** of a number in base 10 is the exponent to which ten must be raised in order to produce the number.

- ▶ If  $x = 10^y$ , then  $y = \log_{10}(x)$  it is called the *logarithm in base 10* of  $x$
- ▶ Logarithms reduce multiplication to simple addition, because  $\log(a \times b) = \log(a) + \log(b)$

8

For more details see: <http://en.wikipedia.org/wiki/Logarithm>

Question: at what point will the line cross 2? (Answer: at one hundred.  $100 = 10^2$ , so  $\log_{10}(100) = 2$ )

Keep in mind that there are also other bases for the logarithms, like the number  $e$  which is used in neperian logarithms (information theory).



# Definition of dB

- ▶ The definition of the decibel uses a logarithm to allow very large or very small *relations* to be represented with a conveniently small number.
- ▶ Let assume we are interested in the ratio between two values  $a$  and  $b$ .
- ▶  $\text{ratio} = a/b$
- ▶ In dB the ratio is defined as:
- ▶  $\text{ratio}_{[\text{dB}]} = 10 \log_{10} (a/b)$
- ▶ It is a dimensionless, relative measure (*a relative to b*)

# Definition of dB

- ▶  $\text{ratio} = 10 \log_{10}(a/b)$
- ▶ What if we now use a value of  $a$  that is 10 times bigger?
- ▶  $\text{newratio} = 10 \log_{10}(10a/b)$

Remember  $\log(a \times b) = \log(a) + \log(b)$

$$\begin{aligned} &= 10 [\log_{10}(10) + \log_{10}(a/b)] \\ &= 10 \log_{10}(10) + 10 \log_{10}(a/b) \\ &= 10 + \text{ratio} \end{aligned}$$

- ▶ The new value (in dB) is simply *10 plus the old value*, so **the multiplication by ten is now expressed by a simple addition of 10 units.**

# Using dB

Commonly used (and easy to remember) dB values:

+10 dB = 10 times the power

-10 dB = one tenth power

+3 dB = double power

-3 dB = half the power

For example:

some power + 10 dB = 10 times the power

some power - 10 dB = one tenth power

some power + 3 dB = double power

some power - 3 dB = half the power

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Memorize these values! You will use them very often.

# dBm and mW

- ▶ What if we want to measure an absolute power with dB?  
**We have to define a reference.**
- ▶ The reference point that relates the logarithmic dB scale to the linear watt scale may be for example this:

$$1 \text{ mW} \rightarrow 0 \text{ dBm}$$

- ▶ The new **m** in dBm refers to the fact that the reference is one **mW**, and therefore a **dBm** measurement is a measurement of absolute power with reference to 1 mW.

12

What if I want to use a logarithmic scale to express a power?

To do so, I have to use a reference value of power. Let's assume it's 1 mW. So I define the dBm.

Using dBm, the reference is 1 milliwatt.

**0 dBm** is therefore equal to **1 mW**.

Note that it is impossible to represent 0 mW (zero power, i.e. no power at all) using dBm!

# dBm and mW

- ▶ To convert power in mW to dBm:

$$P_{\text{dBm}} = 10 \log_{10} P_{\text{mW}}$$

*10 times the logarithm in base 10 of the “Power in mW”*

- ▶ To convert power in dBm to mW:

$$P_{\text{mW}} = 10^{P_{\text{dBm}}/10}$$

*10 to the power of ( “Power in dBm” divided by 10 )*

# dBm and mW

- ▶ Example: mW to dBm

Radio power: 100mW

$$P_{\text{dBm}} = 10 \log_{10}(100)$$

$$100\text{mW} \rightarrow 20\text{dBm}$$

- ▶ Example: dBm to mW

Signal measurement: 17dBm

$$P_{\text{mW}} = 10^{17/10}$$

$$17\text{dBm} \rightarrow 50 \text{ mW}$$

Example: imagine a radio with an output power of 100mW

Example: imagine a signal measurement of 17 dBm

To find the log or the power, in these formulas, you will probably need a calculator. But there are much simpler ways to do these calculations in your mind, we are going to see them in a few minutes!

# Using dB

- ▶ When using dB, gains and losses are **additive**.

Remember our previous example:

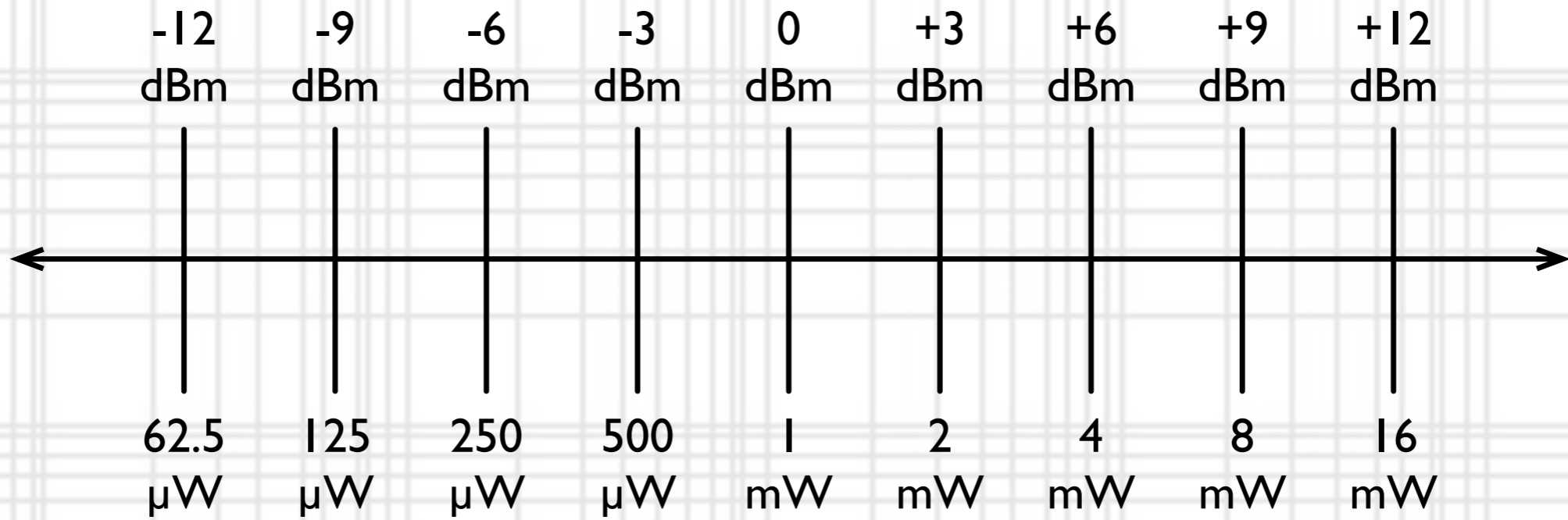
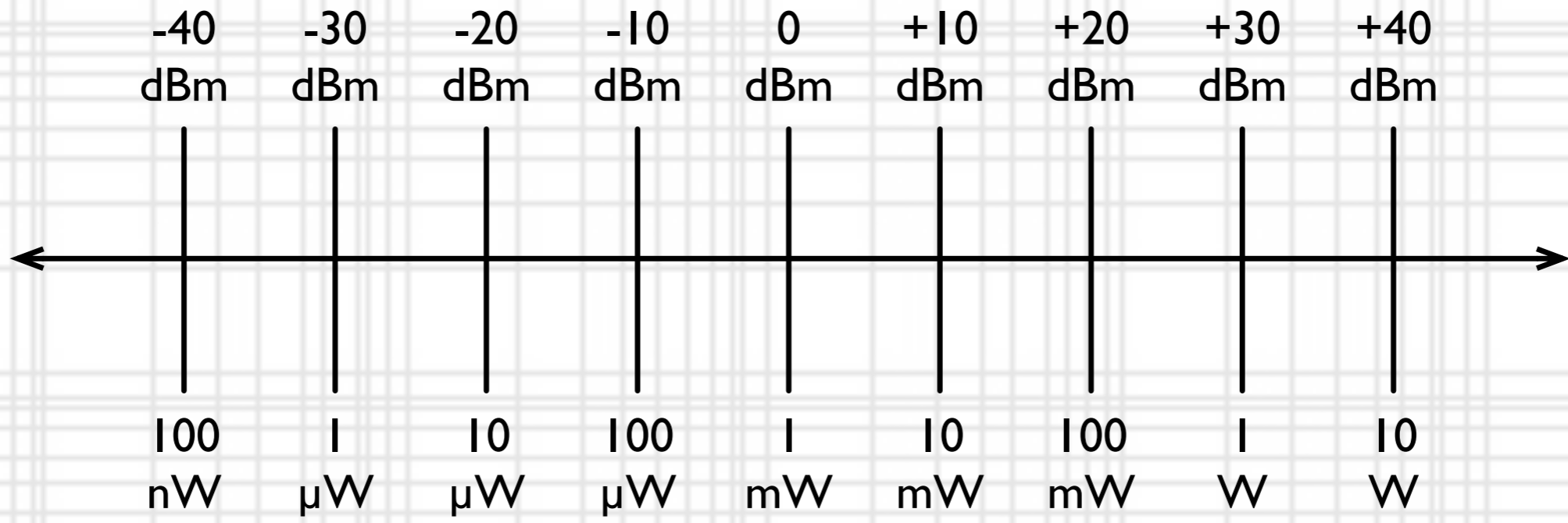
some power + 10 dB = 10 times the power  
some power - 10 dB = one tenth power  
some power + 3 dB = double power  
some power - 3 dB = half the power

You can now imagine situations in which:

10 mW + 10 dB of gain = 100 mW = 20 dBm  
10 dBm = 10 mW = one tenth of 100mW  
20 dBm - 10 dB of loss = 10 dBm = 10mW  
50 mW + 3 dB = 100 mW = 20 dBm  
17 dBm + 3 dB = 20 dBm = 100 mW  
100mW - 3 dB = 50 mW = 17 dBm

Using dB makes the math much simpler, specially when you have to deal with gains and losses.  
Explain all the examples step by step, using the relations above and the fact that 100mW = 20dBm (seen before).  
You can give even more examples now, or ask the students to do the calculation for more examples.

# Using dB



This graph is an easy reference for converting from dBm to mW and vice-versa.



# dB and milliwatts

It is easy to use dB to simplify the addition of gains and losses, then convert back to milliwatts when you need to refer to the absolute power.

1 mW	=	0 dBm
2 mW	=	3 dBm
4 mW	=	6 dBm
8 mW	=	9 dBm
10 mW	=	10 dBm
20 mW	=	13 dBm
50 mW	=	17 dBm
100 mW	=	20 dBm
200 mW	=	23 dBm
500 mW	=	27 dBm
1000 mW (1W)	=	30 dBm

Students don't need to memorize this table, they only have to remember the meaning of 3 and 10 dB and then can easily calculate all other cases, you can show some examples now!

# Simple dB math

How much power is 43 dBm?

- ▶ +43 dBm is 43 dB relative to 1 mW
- ▶ 43 dB = 10 dB + 10 dB + 10 dB + 10 dB + 3 dB

$$\begin{aligned} 1 \text{ mW} \times 10 &= 10 \text{ mW} \\ &\times 10 = 100 \text{ mW} \\ &\times 10 = 1000 \text{ mW} \\ &\times 10 = 10\,000 \text{ mW} \\ &\times 2 = 20\,000 \text{ mW} \\ &= 20 \text{ W} \end{aligned}$$

- ▶ Therefore, +43 dBm = **20 W**

If you have a radio and the specification says that the output power is 43dBm, how much is it in Watts?  
You proceed step by step...

# What about negative values?

Negative doesn't mean bad. ;-)

How much power is -26 dBm?

- ▶ -26 dBm is 1mW (0dBm) “minus” 26 dB
- ▶ -26 dB = -10 dB - 10 dB - 3 dB - 3 dB

$$1 \text{ mW} / 10 = 100 \text{ } \mu\text{W}$$

$$/ 10 = 10 \text{ } \mu\text{W}$$

$$/ 2 = 5 \text{ } \mu\text{W}$$

$$/ 2 = 2.5 \text{ } \mu\text{W} \quad (2.5 * 10^{-6} \text{ W})$$

- ▶ Therefore, -26 dBm = **2.5  $\mu$ W**

19

If you have a received signal measurement of -26 dBm, how much is it in Watts?  
You proceed step by step...

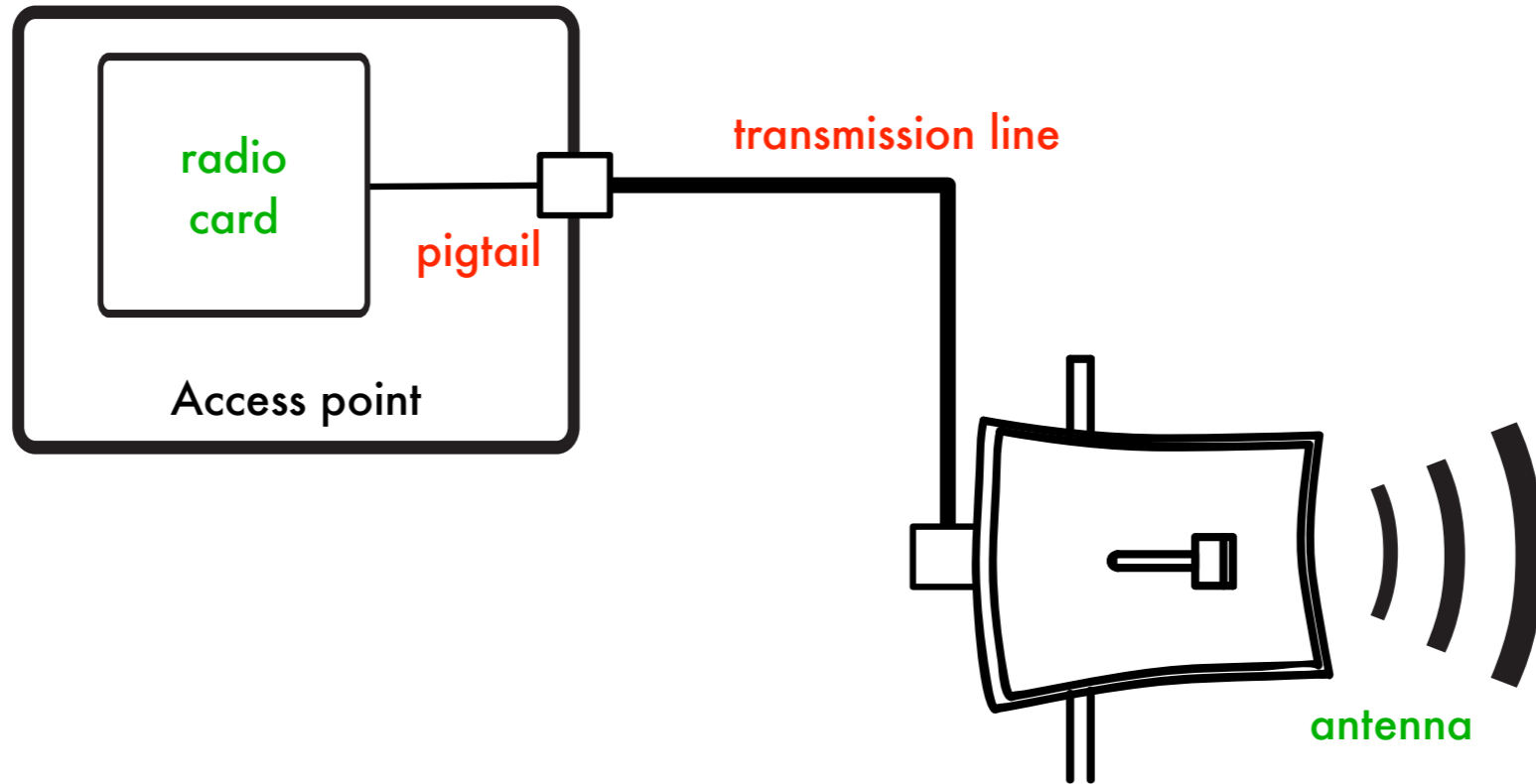
An analogy with altitude may help in explaining negative dBm.

Suppose that in the plans for a building we choose the street level as our reference. Every floor will have a positive height but the basement or cellar will have negative height.

Also keep in mind that using the most common reference for the altitude, meters above the sea level, there are some places (depressions) on earth that have a negative altitude, like Death Valley in U.S.

It is suggested that the instructor makes an example with a building known to the students.

# Example using mW

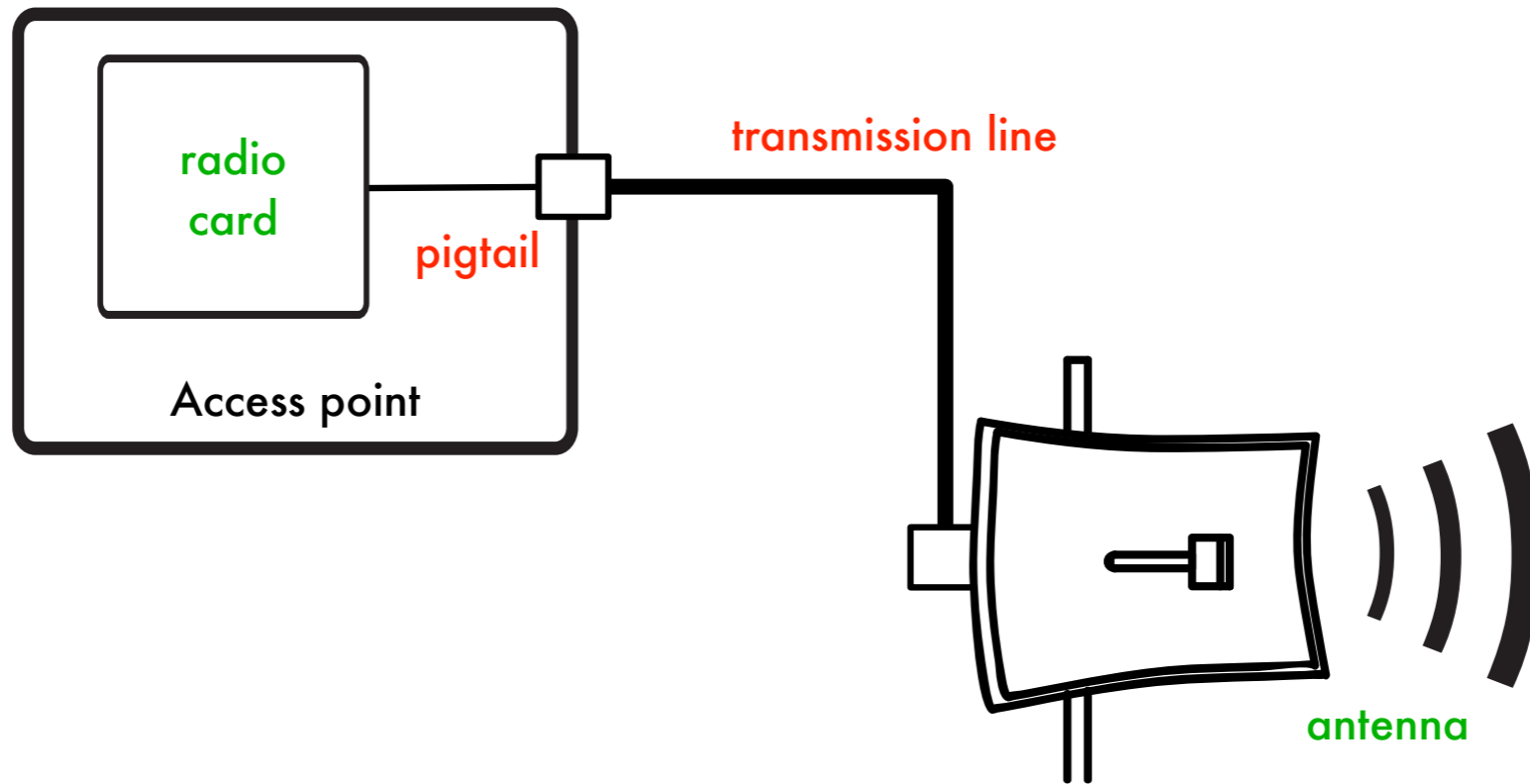


## Using mW

Radio card power	Loss in pigtail	Power leaving Access point	Loss of transmission line	Power entering antenna	Gain of antenna	Power leaving antenna
100 mW	lose half		lose half		16 times the power	
	$100 \text{ mW} / 2$	50 mW				
			$50 \text{ mW} / 2$	25 mW		
					$25 \text{ mW} \times 16$	400 mW

It is difficult to do the math using absolute amounts (mW).

# Example using dB



Using dB

Radio card power	Loss in pigtail	Power leaving Access point	Loss of transmission line	Power entering antenna	Gain of antenna	Power leaving antenna
20 dBm	-3 dB		-3 dB		+12 dBi	
	-3 dB	17 dBm				
			-3 dB	14 dBm		
					+12 dBi	26 dBm (400mW)

Calculation is easier using dB

# Conclusions

- ▶ Using decibels (dB) provides an easier way to make calculations on wireless links.
- ▶ The main advantage of using dB is that gains and losses are **additive**.
- ▶ It is simple to solve radio calculations in your head by using dB instead of using milliwatts.

# Thank you for your attention

For more details about the topics presented in this lecture, please see the book **Wireless Networking in the Developing World**, available as free download in many languages at:

<http://wndw.net/>

