dB math

Training materials for wireless trainers

The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

Goals

- Electromagnetic waves carry power measured in milliwatts.
- Decibels (dB) use a relative logarithmic relationship to reduce multiplication to simple addition.
- You can simplify common radio calculations by using dBm instead of mW, and dB to represent variations of power.
- It is simpler to solve radio calculations in your head by using dB.

Power

- Any electromagnetic wave carries energy - we can feel that when we enjoy (or suffer from) the warmth of the sun. The amount of energy received in a certain amount of time is called ***power***.
- The electric field is measured in V/m (volts per meter), the power contained within it is proportional to the square of the electric field:

$$P \sim E^2$$

- The unit of power is the ***watt*** (***W***). For wireless work, the ***milliwatt*** (***mW***) is usually a more convenient unit.

Gain & Loss

- If the amplitude of an electromagnetic wave increases, its power increases. This increase in power is called a ***gain***.
- If the amplitude decreases, its power decreases. This decrease in power is called a ***loss***.
- When designing communication links, you try to maximize the gains while minimizing any losses.

Intro to dB

- Decibels are a **relative** measurement unit unlike the absolute measurement of milliwatts.
- The ***decibel*** (***dB***) is 10 times the decimal logarithm of the ratio between two values of a variable. The calculation of decibels uses a logarithm to allow very large or very small relations to be represented with a conveniently small number.
- On the logarithmic scale, the reference cannot be zero because the log of zero does not exist!

Why do we use dB?

- Power does not fade in a linear manner, but inversely as the square of the distance.

- You move by **x** and the signal decreases by $\mathbf{1/x^2}$; hence, the **"inverse square law."**

1 meter away → some amount of power
2 meters away → 1/4 power at one meter
4 meters away → 1/16 power at one meter
8 meters away → 1/64 power at one meter

- The fact that exponential relationships are involved in signal strength measurement is one reason why we use a logarithmic scale.

Inverse square law

- The ***inverse square law*** is explained by simple geometry. The radiated energy expands as a function of the distance from the transmitter.

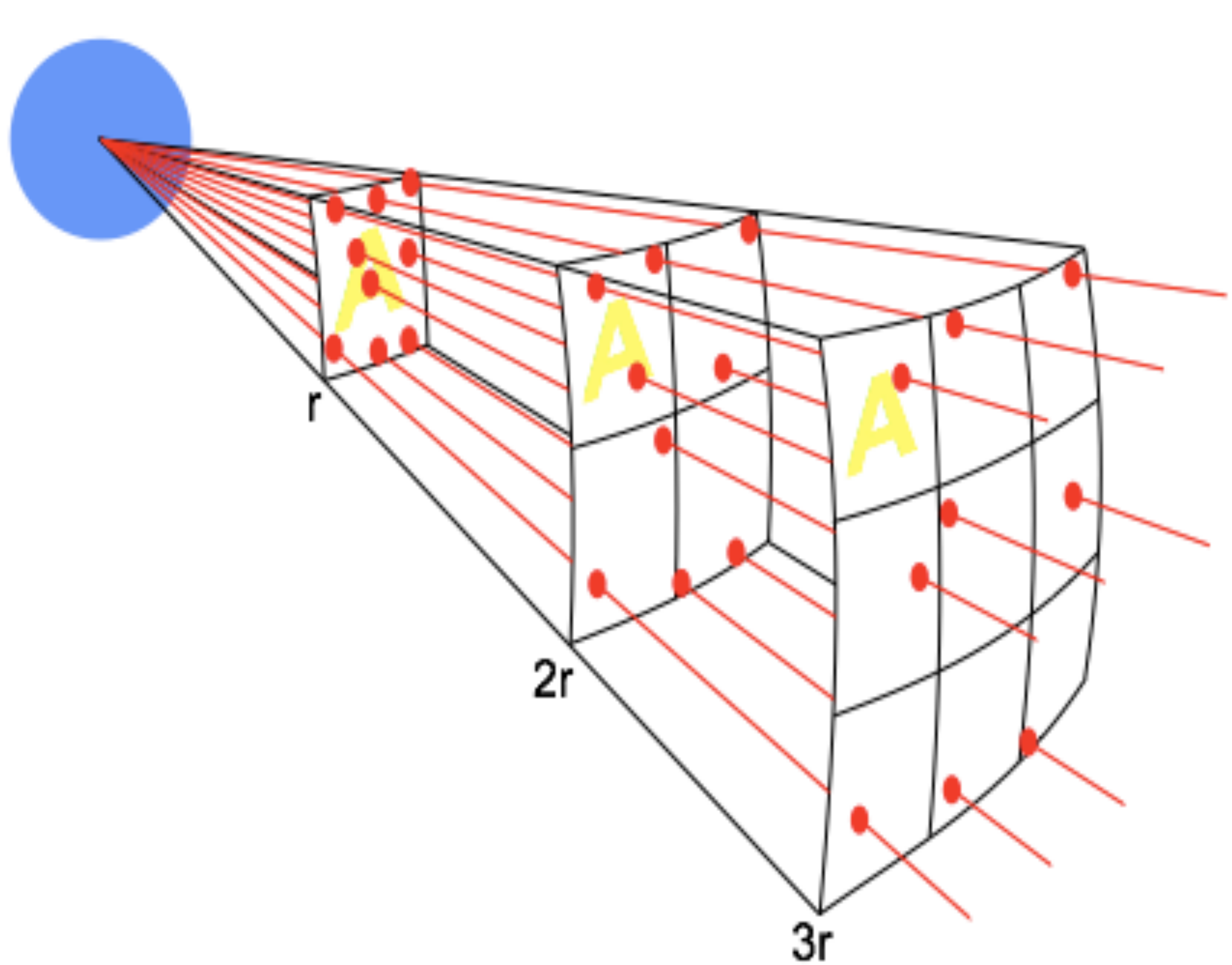
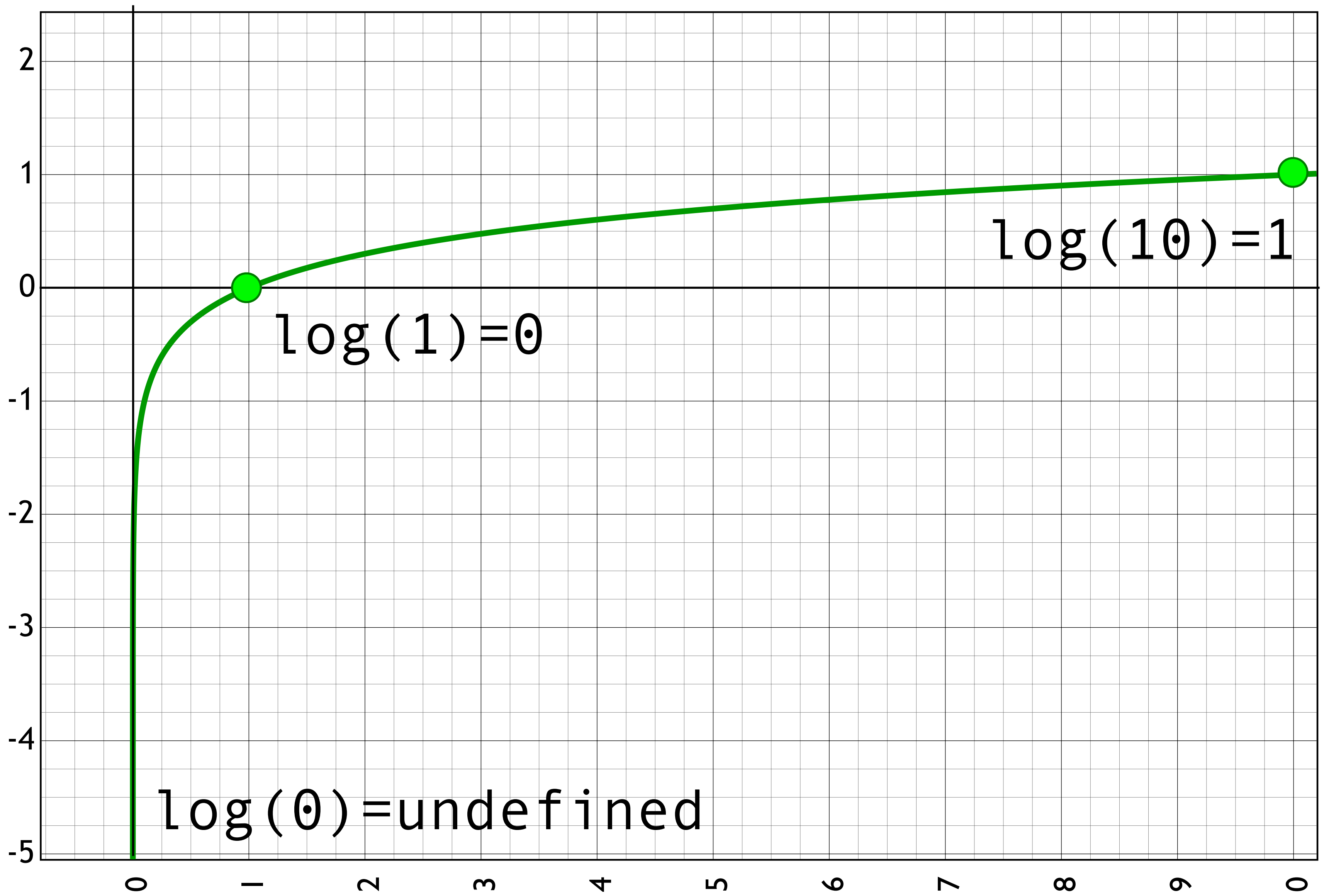


Figure from *http://en.wikipedia.org/wiki/Inverse_square*

A quick review of logarithms



The ***logarithm*** of a number in base 10 is the exponent to which ten must be raised in order to produce the number.

- If $x = 10^y$, then $y = \log_{10}(x)$ — it is called the *logarithm in base 10* of x
- Logarithms reduce multiplication to simple addition, because $\log(a \times b) = \log(a) + \log(b)$

Definition of dB

- The definition of the decibel uses a logarithm to allow very large or very small *relations* to be represented with a conveniently small number.
- Let assume we are interested in the ratio between two values a and b.
- `ratio= a/b`
- In dB the ratio is defined as:
- $\mathtt{ratio}_{[dB]} = 10 \log_{10} (a/b)$
- It is a dimensionless, relative measure (*a relative to b*)

Definition of dB

- $\mathtt{ratio} = 10 \log_{10}(a/b)$
- What if we now use a value of a that is 10 times bigger?
- $\mathtt{newratio} = 10 \log_{10}(10a/b)$

Remember $\log(a \times b) = \log(a) + \log(b)$

$$= 10\,[\log_{10}(10) + \log_{10}(a/b)]$$
$$= 10 \log_{10}(10) + 10 \log_{10}(a/b)$$
$$= 10 + \mathtt{ratio}$$

- The new value (in dB) is simply *10 plus the old value*, so **the multiplication by ten is now expressed by a simple addition of 10 units**.

Using dB

Commonly used (and easy to remember) dB values:

```
+10 dB = 10 times the power
-10 dB = one tenth power
 +3 dB = double power
 -3 dB = half the power
```

For example:

```
some power + 10 dB  = 10 times the power
some power - 10 dB  = one tenth power
some power + 3 dB   = double power
some power - 3 dB   = half the power
```

dBm and mW

- What if we want to measure an absolute power with dB? We **have to define a reference**.
- The reference point that relates the logarithmic dB scale to the linear watt scale may be for example this:

$$1\ \mathrm{mW} \rightarrow 0\ \mathrm{dBm}$$

- The new **m** in dBm refers to the fact that the reference is one **m**W, and therefore a **dBm** measurement is a measurement of absolute power with reference to 1 mW.

dBm and mW

- To convert power in mW to dBm:

$$P_{dBm} = 10 \log_{10} P_{mW}$$

10 times the *logarithm in base 10* of the "Power in mW"

- To convert power in dBm to mW:

$$P_{mW} = 10^{P_{dBm}/10}$$

10 *to the power of* ("Power in dBm" divided by 10)

dBm and mW

- Example: mW to dBm

Radio power: 100mW

$$P_{dBm} = 10 \log_{10}(100)$$

$$100mW \rightarrow 20dBm$$

- Example: dBm to mW

Signal measurement: 17dBm

$$P_{mW} = 10^{17/10}$$

$$17dBm \rightarrow 50\ mW$$

Using dB

- When using dB, gains and losses are ***additive***.

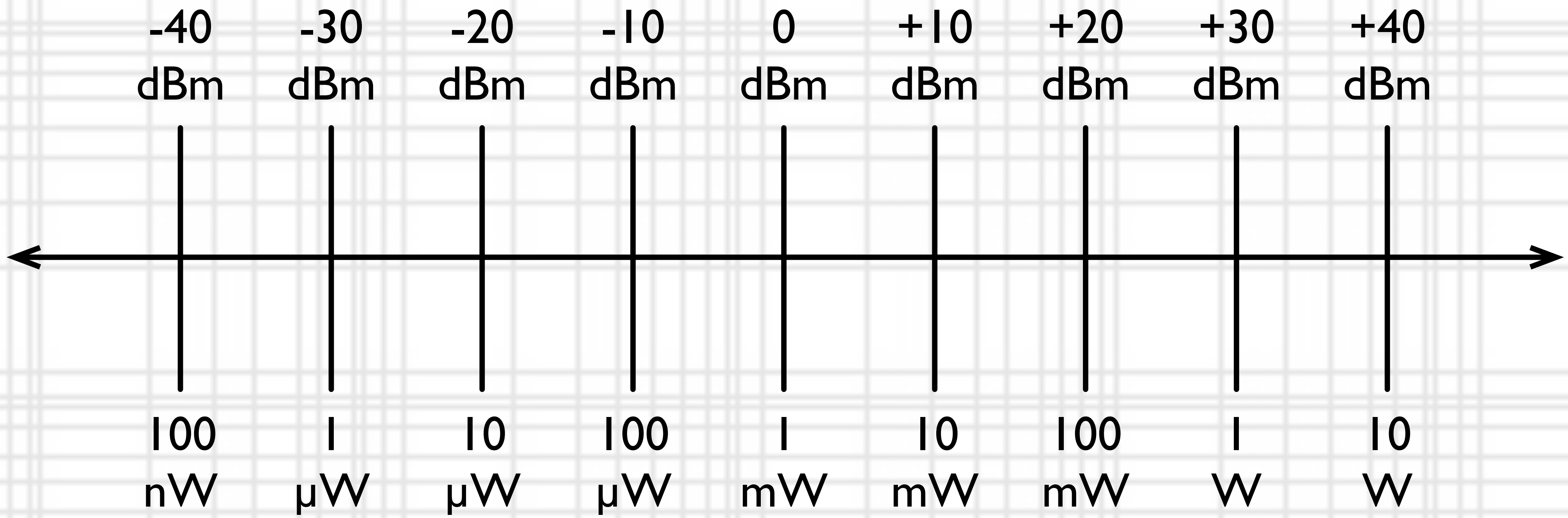
Remember our previous example:

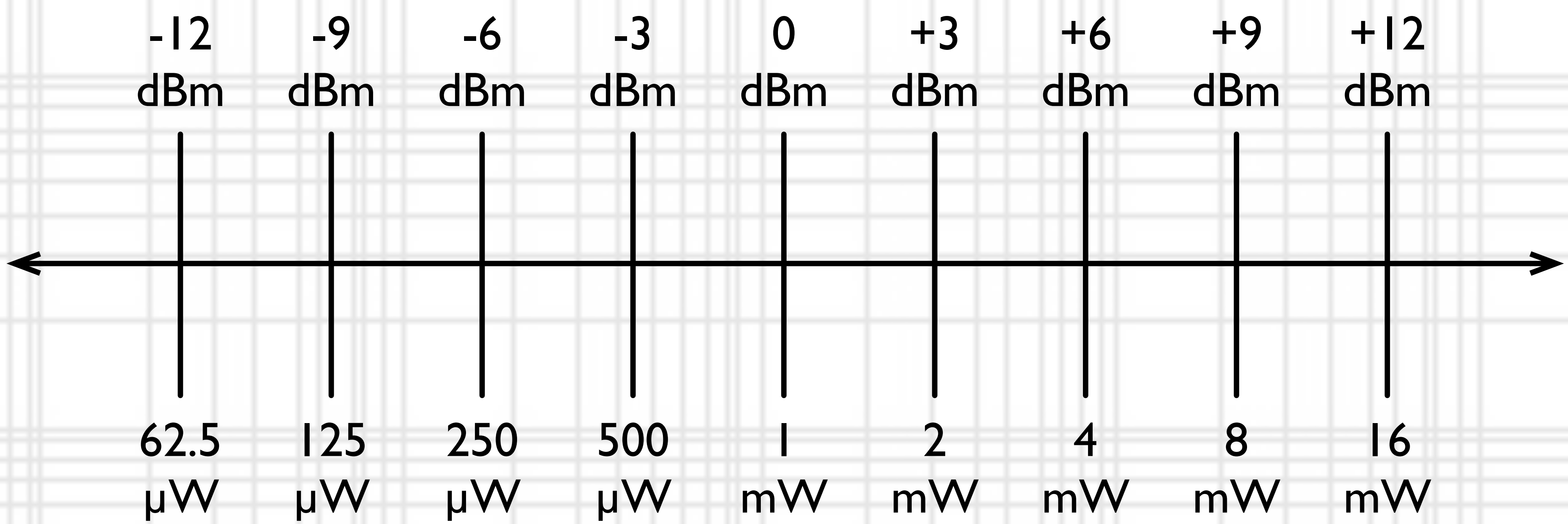
```
some power + 10 dB  = 10 times the power
some power - 10 dB  = one tenth power
some power + 3 dB   = double power
some power - 3 dB   = half the power
```

You can now imagine situations in which:

```
10 mW + 10 dB of gain  = 100 mW = 20 dBm
10 dBm = 10 mW = one tenth of 100mW
20 dBm - 10 dB of loss = 10 dBm = 10mW
50 mW  + 3 dB = 100 mW = 20 dBm
17 dBm + 3 dB = 20 dBm = 100 mW
 100mW - 3 dB = 50 mW  = 17 dBm
```

Using dB


-40 dBm
-30 dBm
-20 dBm
-10 dBm
0 dBm
+10 dBm
+20 dBm
+30 dBm
+40 dBm
100 nW
1 μW
10 μW
100 μW
1 mW
10 mW
100 mW
1 W
10 W


-12 dBm
-9 dBm
-6 dBm
-3 dBm
0 dBm
+3 dBm
+6 dBm
+9 dBm
+12 dBm
62.5 μW
125 μW
250 μW
500 μW
1 mW
2 mW
4 mW
8 mW
16 mW

dB and milliwatts

It is easy to use dB to simplify the addition of gains and losses, then convert back to milliwatts when you need to refer to the absolute power.

1 mW		=	0 dBm
2 mW		=	3 dBm
4 mW		=	6 dBm
8 mW		=	9 dBm
10 mW		=	10 dBm
20 mW		=	13 dBm
50 mW		=	17 dBm
100 mW		=	20 dBm
200 mW		=	23 dBm
500 mW		=	27 dBm
1000 mW	(1W)	=	30 dBm

Simple dB math

How much power is 43 dBm?

- +43 dBm is 43 dB relative to 1 mW
- 43 dB = 10 dB + 10 dB + 10 dB + 10 dB + 3 dB

```
1 mW  x 10 = 10 mW
      x 10 = 100 mW
      x 10 = 1000 mW
      x 10 = 10 000 mW
      x 2  = 20 000 mW
           = 20 W
```

- Therefore, +43 dBm = ***20 W***

What about negative values?

Negative doesn't mean bad. ;-)

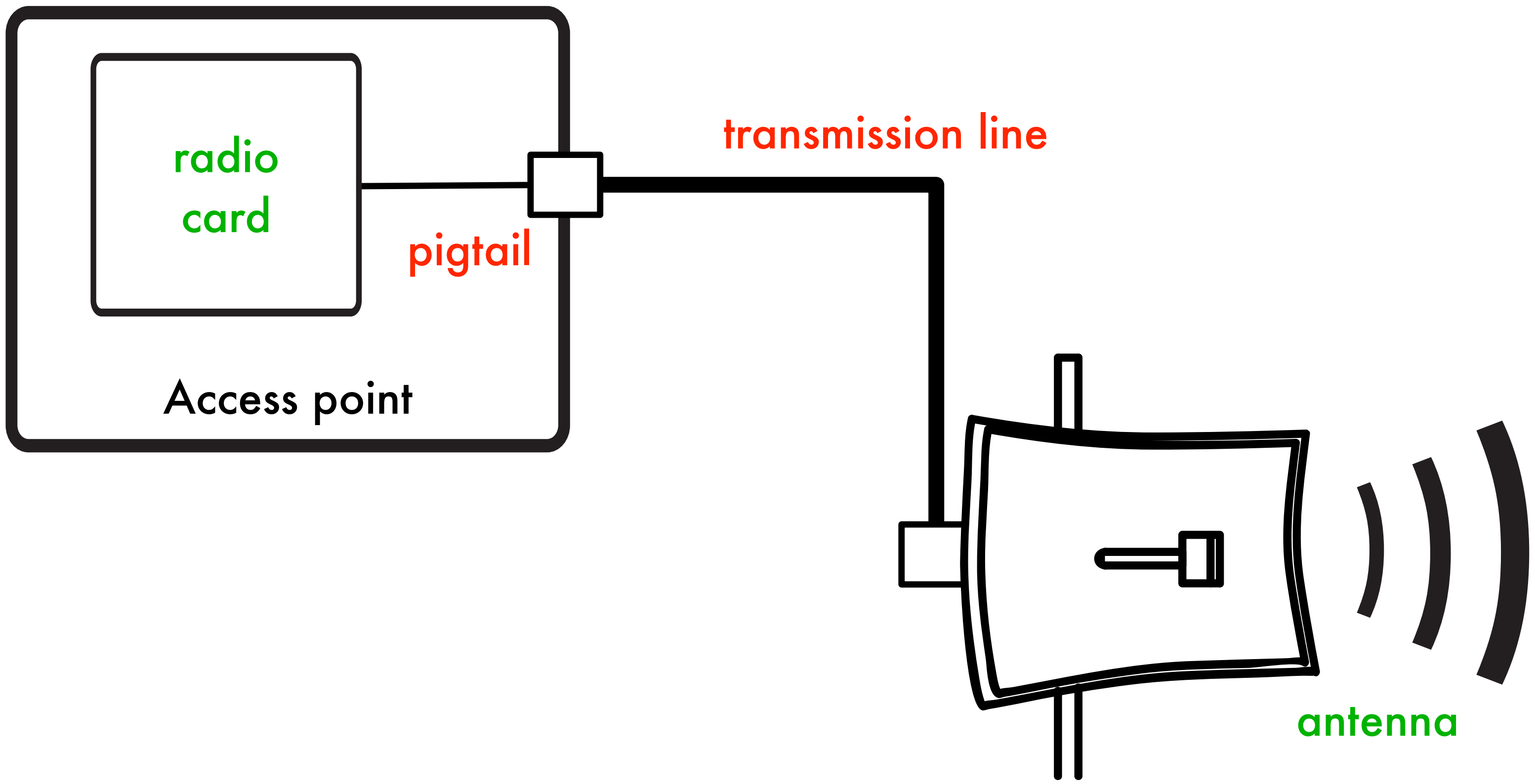
How much power is -26 dBm?

- -26 dBm is 1mW (0dBm) "minus" 26 dB
- -26 dB = -10 dB - 10 dB - 3 dB - 3 dB

```
1 mW / 10 = 100 µW
     / 10 = 10 µW
     / 2  = 5 µW
     / 2  = 2.5 µW (2.5*10^-6 W)
```

- Therefore, -26 dBm = **2.5 µW**

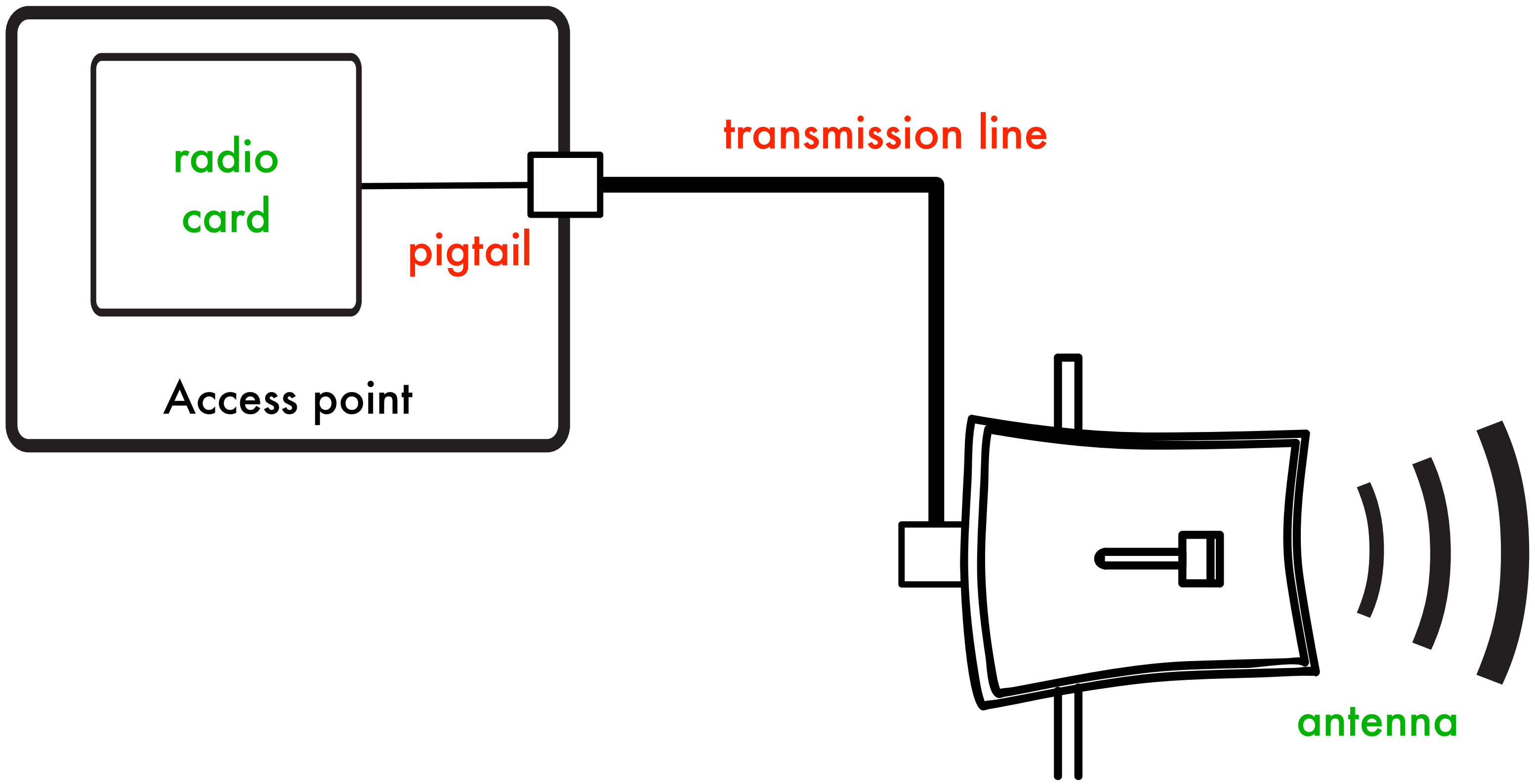
Example using mW



Using mW

Radio card power	Loss in pigtail	Power leaving Access point	Loss of transmission line	Power entering antenna	Gain of antenna	Power leaving antenna
100 mW	loose half		loose half		16 times the power	
	100 mW / 2	50 mW				
			50 mW / 2	25 mW		
					25 mW x 16	400 mW

Example using dB



Using dB

Radio card power	Loss in pigtail	Power leaving Access point	Loss of transmission line	Power entering antenna	Gain of antenna	Power leaving antenna
20 dBm	-3 dB		-3 dB		+12 dBi	
	-3 dB	17 dBm				
			- 3 dB	14 dBm		
					+ 12 dBi	26 dBm (400mW)

Conclusions

- Using decibels (dB) provides an easier way to make calculations on wireless links.
- The main advantage of using dB is that gains and losses are ***additive***.
- It is simple to solve radio calculations in your head by using dB instead of using milliwatts.

Thank you for your attention

For more details about the topics presented in this lecture, please see the book ***Wireless Networking in the Developing World***, available as free download in many languages at:

http://wndw.net/