Transmission Lines

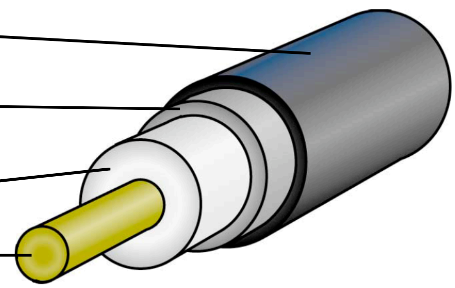
Introduction

The transmitter that generates the RF power to drive the antenna is usually located at some distance from the antenna terminals. The connecting link between the two is the *RF transmission line*. Its purpose is to carry RF power from one place to another, and to do this as efficiently as possible. From the receiver side, the antenna is responsible for picking up any radio signals in the air, and passing them to the receiver with the minimum amount of distortion so that the radio has its best chance to decode the signal. For these reasons, the RF cable has a very important role in radio systems: it must maintain the integrity of the signals in both direction.

There are two main categories of transmission lines: *cables* and *waveguides*.

Cables

RF cables are, for frequencies higher than HF, almost exclusively *coaxial cables* (or "coax" for short, derived from the words "of common axis"). Coax cables have a core wire, surrounded by a non-conductive material (which is called dielectric or insulation), and then surrounded by an encompassing shielding which is often made of braided wires. The dielectric keeps the core and the shielding apart. Finally, the coax is protected by an outer shielding which will generally be a PVC material. The inner conductor carries the RF signal and the outer shield is there to keep the RF signal from radiating to the atmosphere and to stop outside signals from interfering with the signal carried by the core. Another interesting fact is that the electrical signal always travels along the outer layer of the central conductor: the larger the central conductor, the better signal will flow. This is called the “skin effect”.



In the following table the diameters of Core, Dielectric, Shield and Jacket of the most popular Coax cables can be found.

Cable Type	Core (mm)	Dielectric (mm)	Shield (mm)	Jacket (mm)
RG-58	0.9	2.95	3.8	4.95
RG-213	2.26	7.24	8.64	10.29
LMR-400	2.74	7.24	8.13	10.29
3/8" LDF	3.1	8.12	9.7	11

Even though the coaxial construction is good at containing the signal on the core wire, there is some resistance to the electrical flow: as the signal travels down the core, it will fade away. This fading is known as attenuation, and is measured in dB/m. The rate of attenuation is a function of the signal frequency and the physical construction of the cable itself., and a table of these values can be found in the next chapter. Obviously, we need to minimize the cable attenuation as much as possible, keeping the cable very short and using high quality cables.

Practical Hints: How to choose the proper cable

- "The shorter the better!": the first rule when you have to place a cable is to try to keep it as short as possible. The power loss is not linear, so doubling the cable length means that you are going to loose much more than twice the power. In the same way, halfening the cable length gives you more than twice the power at the antenna. The best solution is to place the transmitter as close as possible to the antenna, even when this means placing it on the top of a mast.

- “The cheaper the worse!”: the second golden rule is that the money you invest in buying a good quality cable is a bargain. Cheap cables are intended to be used at low frequencies, not higher than VHF. Microwaves require the highest quality cables, all other options are nothing else than a dummy load.

- always avoid *RG-58*. It is intended for thin Ethernet networking, CB or VHF radio, not for microwave.

- always avoid *RG-213*. It is intended for CB and HF radio. The cable diameter does not imply a high quality, or low attenuation.

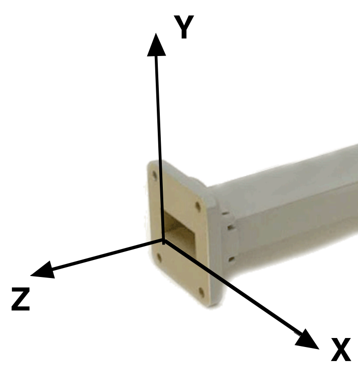
- always use “*Heliax*” (also called “*Foam*”) cables for connecting the transmitter to the antenna, and Semi-rigid cables to interconnect the other devices in the RF chain (i.e. instrumentation). Semi-rigid cables consist of a solid inner conductor (usually copper or silver/copper-plated steel), a solid PTFE dielectric and a solid copper outer pipe. They are sealed, and their rigid construction means that they cannot flex. However the loss is still relatively high due to the small diameter, and they are therefore rarely used as a feeder to an antenna. Heliax cables are basically larger diameter flexible versions of Semi-rigid cables, with a corrugated solid outer conductor to enable them to flex more easily. They can be built in two ways: using air or foam as dielectric. The first solution is the most expensive, guarantees the minimum loss but is very difficult to handle. The second solution is more lossy but less expensive and easier to install. A special procedure is required when soldering connectors, in order to maintain the foam dielectric dry and uncorrupted.

- never step over a cable, never bend it too much, never try to unplug a connector pulling directly the cable. All those behaviors may change the mechanical characteristic of the cable and therefore its impedance, shortcut the inner conductor with the shield, or even break the line. Those problems are difficult to track and recognize and can lead to unpredictable behavior of the radio link.

Waveguides

Above 2 GHz, the wavelength is short enough to allow practical, efficient energy transfer by different means. A waveguide is a conducting tube through which energy is transmitted in the form of electromagnetic waves. The tube acts as a boundary that confines the waves in the enclosed space. The skin effect prevents any electromagnetic effects from being evident outside the guide. The electromagnetic fields are propagated through the waveguide by means of reflections against its inner walls, which are considered perfect conductors. The intensity of the fields is greatest at the center along the X dimension, and must diminish to zero at the end walls because the existence of any field parallel to the walls at the surface would cause an infinite current to flow in a perfect conductor. Waveguides, of course, cannot carry RF in this fashion.

The X, Y and Z dimensions of a rectangular waveguide can be seen in the following figure:



There are an infinite number of ways in which the electric and magnetic fields can arrange themselves in a waveguide for frequencies above the low cutoff frequency. Each of these field configurations is called a *mode*. The modes may be separated into two general groups. One group, designated *TM (Transverse Magnetic)*, has the magnetic field entirely transverse to the direction of propagation, but has a component of the electric field in the direction of propagation. The other type, designated *TE (Transverse Electric)* has the electric field entirely transverse, but has a component of magnetic field in the direction of propagation. TM waves are sometimes called E waves, and TE waves are sometimes called H waves, but the TM and TE designations are preferred. The mode of propagation is identified by the group letters followed by two subscript numerals. For example, TE_{10}, TM_{11}, etc. The number of possible modes increases with the frequency for a given size of guide, and there is only one possible mode, called the *dominant mode*, for the lowest frequency that can be transmitted. In a rectangular guide the critical dimension is X. This dimension must be more than 0.5 λ at the lowest frequency to be transmitted. In practice, the Y dimension usually is made about equal to 0.5 X to avoid the possibility of operation in other than the dominant mode. Cross-sectional shapes other than the rectangle can be used, the most important being the circular pipe. Much the same considerations apply as in the rectangular case. Wavelength dimensions for rectangular and circular guides are given in the following table, where X is the width of a rectangular guide and r is the radius of a circular guide. All figures apply to the dominant mode.

Type of Guide	Rectangular	Circular
Cutoff Wavelength	2X	3.41r
Longest Wavelength transmitted with little attenuation	1.6X	3.2r
Shortest Wavelength before next mode becomes possible	1.1X	2.8r

Energy may be introduced into or extracted from a waveguide by means of either an electric or magnetic field. The energy transfer frequently is through a coaxial line. Two possible methods for coupling to a coaxial line are using the inner conductor of the coaxial line or through a loop. A probe which is simply a short extension of the inner conductor of the coaxial line can be oriented so that it is parallel to the electric lines of force. A loop can be arranged so that it encloses some of the magnetic lines of force. The point at which maximum coupling is obtained depends upon the mode of propagation in the guide or cavity. Coupling is maximum when the coupling device is in the most intense field. If a waveguide is left open at one end it will radiate energy. This radiation can be enhanced by flaring the waveguide to form a pyramidal horn antenna.

Connectors and Adapters

Connectors allow a cable to be connected to another cable or to a component of the RF chain. There is a wide variety of fittings and connectors designed to go with various sizes and types of coaxial lines. We will describe some of the most popular ones.

BNC connectors were developed in the late 40s, and BNC stands for Bayonet Neill Concelman after Amphenol's engineer Carl Concelman. The BNC product line is a miniature quick connect/disconnect connector. It features two bayonet lugs on the female connector, and mating is achieved with only a quarter turn of the coupling nut. BNC's are ideally suited for cable termination for miniature to subminiature coaxial cable (RG - 58 to RG - 179, RG - 316, etc.). They have acceptable performance up to few GHz.

Type N (Navy) connectors were originally developed during the Second World War. They are usable up to 18 Ghz, and very common for microwave and available for almost all types of cable. Both the plug/cable and plug/socket joints are waterproof, providing an effective cable clamp.

SMA is an acronym for SubMiniature version A and was developed in the 60s. 50 Ω SMA connectors are precision, subminiature units that provide excellent electrical performance up to 18 GHz. These high-performance connectors are compact in size and mechanically have outstanding durability.

The *SMB* name derives from SubMiniature B, and it is the second subminiature design. The SMB is a smaller version of the SMA with snap-on coupling. It provides broadband capability through 4 GHz with a snap-on connector design.

MCX connectors were introduced in the 80s. While the MCX uses identical inner contact and insulator dimensions as the SMB, the outer diameter of the plug is 30% smaller than the SMB. This series provides designers with options where weight and physical space are limited. MCX provides broadband capability though 6 GHz with a snap-on connector design.

The *MMCX* series is also called MicroMate™. It is one of the smallest RF connector line and was developed in the 90s. MMCX is a micro-miniature connector series with a lock-snap mechanism allowing for 360 degrees rotation enabling flexibility.

Adapters, which are also called coaxial adapters, are short, two-sided connectors which are used to join two cables or components which cannot be connected directly. In particular, there are two kinds of a adapters. The first one are adapters used to fit connectors of different types. For example an adapter can be used to connect a SMA connector to a BNC one.

Another kind of adapter is the one used to fit connectors of the same type but which cannot be directly joined because of their gender. For example a very useful adapter is the one which enables to join two Type N connectors, having socket (female) connectors on both sides. In this figure, one of these adapters is shown.

Practical Hints: How to choose the proper connector

- "The gender question": connector do have a well-defined gender. Usually cables have male connectors at both sides, while RF devices (i.e. transmitters and antennas) have female connectors. Devices as directional couplers and line-through measuring devices may have both male and female connectors. Moreover, connectors have their own threading: it is usually right-hand, but left-hand thread exists as well, especially for WiFi devices due to some US regulations. Be careful about it.

- "The less the best!": try to minimize the number of connectors/adapters in the RF chain, because each of them introduces some loss (even some dB for each connection!).

- "Buy, don't build!": if possible, buy cables that are already terminated with the connectors you need. Soldering connectors is not an easy task, and to do this job properly is almost impossible for small connectors as MCX and MMCX. Even terminating "Foam" cables is not an easy task.

- don't use BNC for 2.4GHz or higher, use N type connectors (or SMA, SMB, MCX, etc.).

- microwave connectors are precision-made parts, and can be easily damaged by mistreatment. As a general rule, if the connectors have threaded sleeves, you should rotate these to tighten, leaving the rest of the connector (and cable) stationary. If other parts of the connector are twisted while tightening or loosening, damage can easily occur.

- never step over connectors, or drop connectors on the floor when disconnecting cables (this happens more often than what you may imagine, especially when working on a mast over a roof).

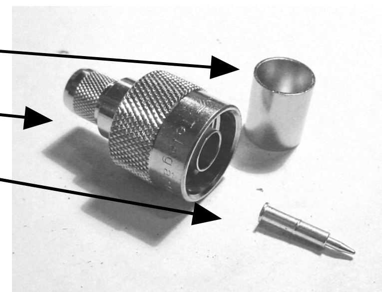
- never use tools like pliers to tighten connectors, use your hands. When working outside, remember that metals expand at high temperatures and reduce their size at low temperatures: a very tightened connector in the summer can even break in winter.

Experiment 1: How to put together an N Type connector

As an example, we will learn how to terminate an RG 213 like coaxial cable with an N type male connector.

Equipment Required:

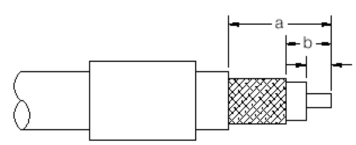
- the connector. It is provided in three parts: the outer ring (for crimping), the connector body and the inner pin (which can be soldered or crimped).
Other versions of the connector exist, intended for soldering;



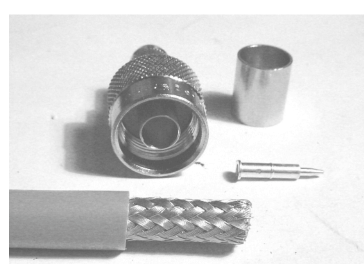
- the cable. Remember to add to the required length of the cable at least 2 cm at each end, for the connectors;
- a cutter and/or a pair of scissors;
- a soldering iron with a small tip;
- some solder (1 mm of thickness, 60%-40%);
- a crimping tool for RG 213.

Procedure:

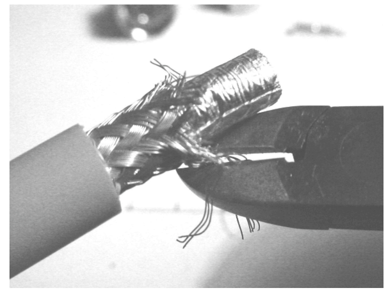
- Strip cable jacket, braid, and dielectric to the proper dimensions (there should be an instruction sheet included with the connector). All cuts are to be sharp and square.



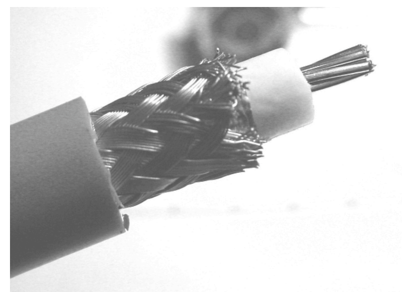
1) Trim the jacket with the cutter for a length of ‘a’ mm. Don’t put too much pressure in cutting the jacket, proceed gradually in order not to nick the braid.



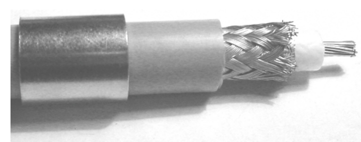
2) Cut the braid with the scissors for a length of ‘b’ mm. Good quality cables have two layers of braid, with a thin metallic sheet inside. Cut also this sheet at the same length.



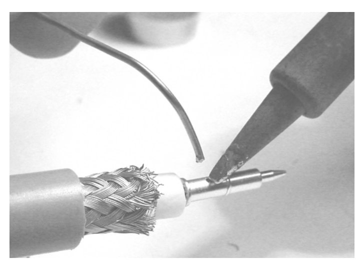
3) Cut the dielectric with the cutter for a length of ‘c’ mm. Do not nick the center conductor. Tinning of center conductor is not necessary if contact is to be crimped. For solder method, tin center conductor avoiding excessive heat.



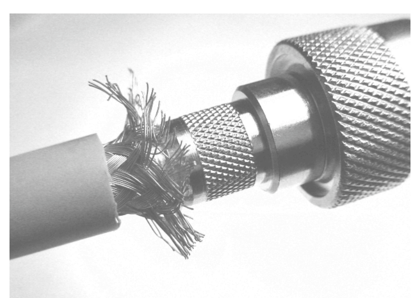
4) Slide outer ring onto cable as shown. Do not comb out braid.



5) Place the central pin on cable’s center conductor so it butts against cable dielectric. Center conductor should be visible through inspection hole in the central pin. Solder (or crimp) the pin in place. Do not get any solder on outside surface of the pin. Avoid excessive heat to prevent swelling of dielectric.

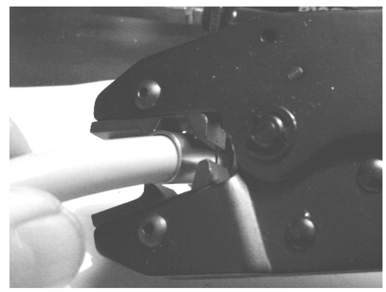


6) Insert the cable into the connector body so the inner portion slides under the braid. Push cable assembly forward until the pin snaps into place. You should hear a ‘click’ sound.

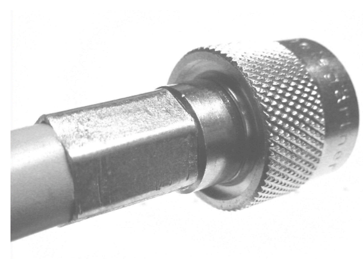


7) Slide outer ring over braid and up against connector body.

8) Crimp outer ring using the crimping tool. It is a one-shot operation, so be careful in positioning the tool properly around the ring and press strongly. If necessary, you can crimp the ring few more times.



9) The final result should look like the picture.



10) Check with an Ohmmeter to assure you don’t have a short circuit between the central pin and the connector body.

Impedance and Impedance Matching

Transmission system behavior differs at low and high frequencies, and the different behaviors are usually described in terms of lumped-constant and distributed-constant systems. Lumped-constant circuits involve components (coils, resistors, capacitors, etc.) whose physical dimensions are much less than the wavelength of the propagating electromagnetic wave and which can be located at discrete points. When circuit components and connecting wires are of dimensions comparable to a wavelength of the propagating electromagnetic wave, then the circuit components and the wires effectively become distributed constants. We may then think of a line as begin composed of a series of small inductors and capacitors, where each coil is the inductance of an extremely small section of wire, and the capacitance is that existing between the same two sections. Each series inductor acts to limit the rate at which current can charge the following shunt capacitor, and in so doing it establishes a very important property of a transmission line, its characteristic impedance. This is abbreviated by convention as Z_0.

The value of a the characteristic impedance is equal to $\sqrt{\frac{L}{C}}$ in a perfect line, one in which the conductors have no resistance and there is no leakage between them. L and C are respectively the inductance and capacitance per unit of length of line. The inductance decreases with increasing conductor diameter, and the capacitance decreases with increasing spacing between the conductors. Hence a line with closely spaced large conductors has a low characteristic impedance, while one with widely spaced thin conductors has a high one. Typical coaxial lines can have characteristic impedance's ranging from $30\,\Omega$ to $100\,\Omega$, but most common impedance values for coaxial cables are $50\,\Omega$ and $75\,\Omega$. Physical constraints on practical wire diameters and spacing limit Z_0 values to these ranges. The $50\,\Omega$ RG-58 cable was developed during World War II to connect antennas which had an impedance of $50\,\Omega$.

A line terminated in a purely resistive load equal to the characteristic line impedance is said to be *matched*. In a matched transmission line, the power is transferred outward from the source until it reaches the load, where it is completely absorbed. Thus with either an infinitely long line or a matched one, the impedance presented to the source of power is the same, regardless of the line length: it is equal to the characteristic impedance of the line. The current in such a line is given by the applied voltage divided by the characteristic impedance, according to Ohm's law.

If the terminating resistance R is not equal to Z_0, then the line is said to be *mismatched*. The more the R differs from Z_0, the greater the mismatch. The power reaching R is not totally absorbed, as it was when R was equal to Z_0, because R requires a voltage to current ratio that is different from the one traveling along the line. The result is that R absorbs only part of the power reaching it, the *incident or direct power*. The remainder goes back along the line toward the source, and it is knows as the *reflected power*. The greater the mismatch, the larger the percentage of the incident power that is reflected. In the extreme case when R is zero (a short circuit) or infinity (an open circuit), all of the power reaching the end of the line is reflected back toward the source. When there is a mismatch, power is transferred in both directions along the line. The voltage to current ratio must be the same for the reflected power and for the incident one, because this ratio is determined by the Z_0 of the line. The actual voltage at any point along the line is the vector sum of the incident voltage and of the reflected voltage, taking into account the phases of each component. The same is true for the current. The effect of the incident and reflected components on the behavior of the line can be understood by considering two limiting cases: the short-circuited line and the open-circuited line. If the line if short-circuited, the voltage at the end of the line must be zero. Thus the incident voltage must disappear at the short. Is can do this only if the reflected voltage is opposite in phase and of the same amplitude. The current, however, does not disappear in the short circuit. The incident current flows through the short and there is in addition the reflected component in phase with it and of the same amplitude. The reflected voltage and current must have the same amplitudes of the incident ones, because no power is dissipated in the short circuit. Reversing the phase of either the current or voltage reverses the direction of the power flow. If the line is open-circuited, the current must be zero at the end of the line. In this case the reflected current must be opposite in phase with the incident current, and with the same amplitude. The reflected voltage must be in phase with the incident voltage, and must have the same amplitude. When there is a finite value of resistance at the end of the line, only part of the power is reflected. That is, the reflected voltage and current are smaller than the incident ones. The amplitudes of the two components are therefore not equal, but the resultant current and voltage are in phase in R because R is a pure resistance.

The ratio of the reflected voltage at a given point on a transmission line to the incident voltage is called the *voltage reflection coefficient*. The voltage reflection coefficient is also equal to the ratio of the incident and reflected currents.

$$\rho = \frac{E_r}{E_i} = \frac{I_r}{I_i}$$

where

- ρ is the reflection coefficient
- E_r is the reflected voltage
- E_i is the incident voltage
- I_r is the reflected current
- I_i is the incident current

The reflection coefficient is determined by the relationship between the line's characteristic impedance and the actual load at the end of the line. In most cases, the load is not entirely resistive. It is a complex impedance, consisting of a resistance in series with a reactance. The reflection coefficient is thus a complex quantity, having both amplitude and phase. It can be designated with the letter ρ or with the letter Γ.

The relationship between R_a, the load resistance, X_a, the load reactance, Z_0, the line characteristic impedance with real part R_0 and reactive part X_0 and the complex reflection coefficient is given by:

$$\rho = \frac{Z_a - Z_0}{Z_a + Z_0} = \frac{(R_a \pm jX_a) - (R_0 \pm jX_0)}{(R_a \pm jX_a) + (R_0 \pm jX_0)}$$

For most transmission lines the characteristic impedance is almost completely resistive, meaning that $Z_0 = R_0$ and $X_0 = 0$. The magnitude of the complex reflection coefficient then simplifies to:

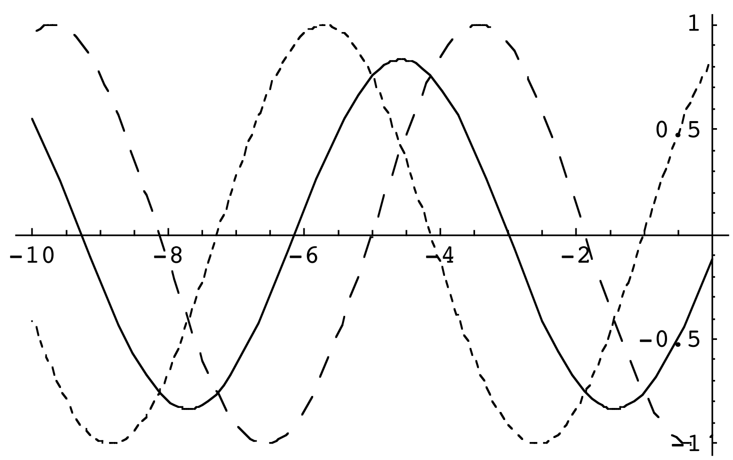
$$|\rho| = \sqrt{\frac{(R_a - R_0)^2 + X_a^2}{(R_a + R_0)^2 + X_a^2}}$$

Example: if the characteristic impedance of a coaxial line is 50 Ω and the load impedance is 140 Ω in series with a capacitive reactance of -190 Ω, the magnitude of the reflection coefficient is

$$|\rho| = \sqrt{\frac{(140 - 50)^2 + 190^2}{(140 + 50)^2 + 190^2}} = 0.782$$

If R_a is equal to R_0 and if X_a is zero, the reflection coefficient is also zero: this is the case of the matched line. On the other hand, if R_a is equal to zero, meaning that the load has no resistive part, then the reflection coefficient is equal to 1 regardless of the value of R_0. This means that all the forward power is reflected, since the load is completely reactive.

As a consequence of reflection, a standing wave may be visualized as an interference between the incident signal E_i at a given frequency, traveling in the forward direction, and the signal E_r, at the same frequency, traveling in the reverse direction. At the load, the relationship between the amplitudes of E_r and E_i and the phase angle between them are uniquely determined by the load impedance. The phase angle between E_r and E_i, however, will vary along the line as a function of the distance from the load. A wave is created that oscillates in amplitude but never moves laterally. That is why it is called *standing wave*. In the following figure, the E_r, E_i and the standing wave can be seen. The dashed lines are the E_r and the E_i, while the non dashed one represents the standing wave.



At a position 180° from the load ($\frac{1}{2}\lambda$), the voltage and current must have the same values they do at the load. At a position 90° from the load ($\frac{1}{4}\lambda$), the voltage and current must be inverted: if the voltage is lowest and the current is highest at the load, then at 90° from the load the voltage reaches its highest value and the current reaches its lowest value at the same point. Note that the conditions at 90° also exist at 270°, and the ones at 180° are valid at every point multiple of 180°.

In a matched line, all of the power that is transferred along the line is absorbed in the load if the load is equal to the characteristic impedance. None of the power is reflected back toward the source. As a result, no standing waves will be developed along the line. The voltage along the line is constant, so the matched line is also said the be flat.

The ratio of the maximum voltage, resulting from the interaction of incident and reflected voltages along the line, to the minimum voltage is defined as the *Voltage Standing-Wave Radio (VSWR)* or simply *Standing-Wave Radio (SWR)*. The ratio of the maximum current to the minimum current is the same as the VSWR, so either current or voltage can be measured to determine the standing-wave ratio.

$$SWR = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}}$$

In the case where the load contains no reactance, the SWR is equal to the ratio between the load resistance R and the characteristic impedance of the line. The standing-wave ratio is an index of many of the properties of a mismatched line. The SWR is related to the magnitude of the complex reflection coefficient by the following equation

$$SWR = \frac{1+|\rho|}{1-|\rho|}$$

where we can see that with $|\rho| = 0$ we get SWR=1, so we have maximum transmission, and with $|\rho| = 1$ we get SWR=∞, so we have no transmission. And the reflection coefficient magnitude may be defined from a measurement of SWR as

$$|\rho| = \frac{SWR - 1}{SWR + 1}$$

We may also express the reflection coefficient in terms of forward and reflected power, quantities that can be easily measured using a directional RF wattmeter. The reflection coefficient may be computed as

$$\rho = \sqrt{\frac{P_r}{P_f}}$$

where P_f is the power in the forward wave and P_r is the power in the reflected wave. We can use this equation to calculate the SWR from a measurement of the forward and reflected power

$$SWR = \frac{1+|\rho|}{1-|\rho|} = \frac{1+\sqrt{\frac{P_r}{P_f}}}{1-\sqrt{\frac{P_r}{P_f}}}$$

The relation between the *Return Loss* expresses in dB, which is the amplitude of the reflected wave to the amplitude of the incident wave, and the reflection coefficient is given by:

$$\text{Return Loss (dB)} = -20\log(\rho)$$

The relation between the power ratio and the reflection coefficient is given by:

$$\frac{P_r}{P_f} = 100\rho^2$$

For example, if we measure a return loss of 15 dB, then we can calculate the reflection coefficient as 0.178 and thus the SWR as 1.43.

Attenuation in Transmission Lines

Every transmission line will have some loss, because of the resistance of the conductors and because power is consumed in the dielectric used for insulating the conductors. Power lost in a transmission line is not directly proportional to the line length, but varies logarithmically with the length. For this reason line losses are expressed in terms of decibels per unit length, since the decibel is a logarithmic unit. Calculations are very simple because the total loss in a line is found by multiplying the decibel loss per unit length by the total length of the line.

The power lost in a matched line is called *matched-line loss*. It is usually expressed in decibels per 100 feet. It is necessary to specify the frequency for which the loss applies, because the loss varies with frequency. Conductor and dielectric losses increase with frequency, but not in the same way. The relative amount of each type of loss depends also on the construction on the line, so there is no specific relationship between loss and frequency valid for all types of lines. Actual loss values for practical lines can be found in the following table, expressed in $\frac{dB}{100m}$.

Cable Type	144 MHz	1.2 GHz	2.4 GHz	5.8 GHz
RG-58	20.3	69.2	105.6	169.2
RG-213	9.2	33.1	49.9	93.8
LMR-400	4.9	15.7	22.3	35.4
3/8" LDF	4.3	13.8	19.4	26.6

The power lost in a given line is minimum when the line is terminated in a resistance equal to its characteristic impedance. On non-matched lines there is an additional loss that increases with the increase of the SWR. This is because the effective values of both current and voltage become greater on lines with standing waves. This increase raises the ohmic losses (I^2R) in the conductors and the losses in the dielectric (E^2/R).

The total loss in a line, including matched-line and the additional loss due to standing waves may be calculated as follows

$$\text{Total Loss (db)} = 10\log\left[\frac{\alpha^2 - |\rho|^2}{\alpha(1-|\rho|^2)}\right]$$

where

- $\alpha = 10^{\frac{ML}{10}}$ = matched-line loss ratio
- $|\rho| = \frac{SWR - 1}{SWR + 1}$ = magnitude of reflection coefficient

ML is the matched-line loss for a particular length of the line, expressed in dB, and SWR is the standing-wave ratio at the load end of the line.

Thus the additional loss caused by the standing waves is calculated from

$$\text{Additional Loss (dB)} = \text{Total Loss} - \text{ML}$$

Example: let us consider a RG-213 coaxial cable at 2.4 GHz. The matched-line loss is rated at 49.9 dB per 100 meters. A 10-meter length of RG-213 would then have an overall matched-line loss of

$$\frac{49.9}{100} \times 10 = 4.99\text{dB}$$

If the SWR at the load end of the cable is 4:1, then

$$\alpha = 10^{\frac{4.99}{10}} = 3.155$$
$$|\rho| = \frac{4-1}{4+1} = 0.6$$

The total line loss can then be calculated as

$$\text{Total Loss (dB)} = 10\log\left[\frac{3.155^2 - 0.6^2}{3.155(1-0.6^2)}\right] = 15.5\text{dB}$$

The additional loss due to the SWR of 4:1 is then

15.5-4.99=10.51 dB

If the SWR at the load end of the cable is 2:1, then

$$|\rho| = \frac{2-1}{2+1} = 0.33$$

and the total line loss is then

$$\text{Total Loss (dB)} = 10\log\left[\frac{3.155^2 - 0.33^2}{3.155(1-0.33^2)}\right] = 12.5\text{dB}$$

The additional loss due to the SWR of 2:1 is then

12.5-4.99=7.51 dB

It is often desirable to know the voltages and currents that are developed in a line operating with standing waves. The voltage maximum may be calculated as follows

$$E_{max} = \sqrt{P \times Z_0 \times SWR}$$

where

- E_{max} is the voltage maximum along the line in the presence of standing waves

- P is the power delivered by the source to the line input, expressed in Watts

- Z_0 is the characteristic impedance of the line, expressed in Ohms

- SWR is the SWR at the load

The voltage determined is the RMS value, the voltage that would be measured with an ordinary RF voltmeter. To consider the risk of voltage breakdown, this value should be converted to an instantaneous peak voltage. This is done multiplying the RMS value by $\sqrt{2}$ if we assume the RF waveform to be a sine wave. The resultant value is the maximum possible value that can exist along a line, and it is useful in determining whether a particular line can operate safely with a given SWR.

Example: if 100 Watts of power are applied to a 50 Ω line with a SWR at the load of 4:1, then

$$E_{max} = \sqrt{100 \times 50 \times 4} = 141.4\ V$$

$$E_{max}\ (\text{instantaneous peak}) = 141.1 \times \sqrt{2} = 199.4\ V$$

Since

$$SWR = \frac{E_{max}}{E_{min}}$$

then

$$E_{min} = \frac{E_{max}}{SWR}$$

The current can be found using Ohm's law:

$$I_{max} = \frac{E_{max}}{Z_0} = 2.82 \text{ A}$$

$$I_{min} = \frac{I_{max}}{SWR} = 0.70 \text{ A}$$

It is very useful to relate the value of attenuation with the power lost along a line and with the values of voltage at the input and at the output of the line. We have

$$A(\text{power}) = 10 \log_{10} \frac{P_{out}}{P_{in}}$$

$$A(\text{voltage}) = 20 \log_{10} \frac{V_{out}}{V_{in}}$$

We can also use this equation to calculate the power that is going to be delivered to the load when we have a certain power at the input and a certain value of attenuation.

We can calculate this power as

$$P_{out} = P_{in}\ 10^{\frac{A}{10}}$$

A transmission line can be considered also to be an impedance transformer. A certain value of load impedance, consisting of a resistance and reactance, at the end of a particular transmission line is transformed into another value of impedance at the input of the line. The amount of this transformation is determined by the length of the line, its characteristic impedance, and by the losses in the line. The input impedance of a lossy transmission line of length *L* is calculated using the *Transmission Line Equation*:

$$Z_{in} = Z_0 \frac{Z_L \cosh(\gamma L) + Z_0 \sinh(\gamma L)}{Z_L \sinh(\gamma L) + Z_0 \cosh(\gamma L)}$$

where

- Z_{in} is the complex impedance at the input of the line
- Z_L is the complex load impedance at the end of the line
- Z_0 is the characteristic impedance of the line
- γ is the complex loss coefficient, $\gamma = \alpha + \beta$
- α is the matched line attenuation constant, expressed in nepers/unit length (1 neper=8.688 dB)
- β is the phase constant of line in radians/unit length (2π =one wavelength). In particular:

$$\beta = \frac{2\pi}{VF \times \frac{983.6}{f(MHz)}}$$

where VF is the velocity factor and L is the electrical length in the same units as α and β. The velocity factor is related to the dielectric constant ε by

$$VF = \frac{1}{\sqrt{\varepsilon}}$$

and is usually given by the cable manufacturer. It is an adimensional value always lower than 1 which represents the relation between the propagation speed of an electromagnetic wave in the dielectric and in vacuum. It shows the delay to be expected during signal propagation along the line coming from the usage of the dielectric as a support and electric insulation for the conductors, so its value depends on the dielectric constant.

Example: an antenna terminates a 50 feet long piece of RG-213 coaxial cable, and we want to calculate the impedance value at the input. The antenna is assumed to have an impedance of $43 + j30\Omega$ at 7.15 MHz. Its velocity factor is 0.66. The matched line loss of the line at 7.15 MHz is 0.27 dB/ 100 feet. The characteristic impedance for this type of cable is 50Ω. We must first calculate the values of α and β as:

$$\alpha = \frac{1 \text{ neper} \times 0.27}{8.688} \times \frac{1}{100} = 0.000310773$$

$$\beta = \frac{2\pi}{0.66 \times \frac{983.6}{7.15}} = 0.0692029$$

The complex loss coefficient is then:

$$\gamma = 0.000310773 + j0.0692028$$

We can finally calculate the complex impedance at the input of the line as:

$$Z_{in} = 65.6563 + j33.2804$$

A special case of impedance transformation is a line which is an exact multiple of 1/4 λ. Such a line will have a purely resistive input impedance when the termination is a pure resistance. When the line losses are low, a short circuit as load is seen as an inductor at the input, while an open circuit is seen as a capacitor.

When the line length is an even multiple of 1/4 λ (i.e. a multiple of 1/2 λ), the input resistance is equal to the load resistance, regardless of the line Z_0. A line which is an exact multiple of 1/2 λ simply repeats at its input whatever impedance exists at its output. Sections of lines having such length can be added or removed without changing any of the operating conditions, when the losses are negligible. When the line length is an odd multiple of 1/4 λ the input impedance of the line becomes

$$Z_{in} = \frac{Z_0^2}{Z_l}$$

where Z_{in} is the input impedance and Z_l is the load impedance. If Z_l is a pure resistance, Z_{in} will also be a pure resistance. Rearranging the equation gives

$$Z_0 = \sqrt{Z_{in} \times Z_l}$$

This means that if we have two values of impedance that we wish to match, we can connect them together by a 1/4 λ transmission line having a characteristic impedance equal to the square root of their product. A 1/4 λ transmission line is, in effect, a transformer.

By knowing the parameters Z_0, α and β, one can extract the parameters R (series resistance per unit length of line), L (series inductance per unit length of line), C (shunt capacitance per unit length of line) and G (shunt conductance per unit length of line) which define the transmission line. The equations are the following

$$R = R_0\alpha - x_0\beta\left(\frac{\Omega}{m}\right)$$

$$L = \frac{R_0\beta + x_0\alpha}{\omega}\left(\frac{H}{m}\right)$$

$$G = \frac{R_0\alpha + x_0\beta}{R_0^{\ 2} + x_0^{\ 2}}\left(\frac{\Omega^{-1}}{m}\right)$$

$$C = \frac{R_0\beta - x_0\alpha}{(R_0^{\ 2} + x_0^{\ 2})\omega}\left(\frac{F}{m}\right)$$