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dB: Units & Calculations

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Purpose

- To review the definition and applications of the dB unit

Outline

- What “dB” is and why is it used?
- A few examples
- Some calculations
- Summary

dB

- The ***Decibel***, or 1/10th of Bel (abbreviation 'dB'), is one of most popular unit used in radiocommunications
  - The name is in honor of Alexander Graham Bell (1847-1922), a Scottish-born teacher of the deaf and American inventor of the telephone
- It is applicable to dimension-less physical quantities 'pure numbers' such as gain, loss, protection ratio, etc.

Definition

Let 'x' be the 'magnitude' of a dimensionless variable.

Let 'X' be the 'level' of that variable expressed in dB.

The 'magnitude' and the 'level' are interrelated:

$$X = 10\log_{10}(x) \qquad \text{and} \qquad x = 10^{\left(\frac{X}{10}\right)}$$

Why dB?

- The dB is used rather than arithmetic ratios or percentages because
  - when circuits are connected in tandem, expressions of power level, in dB, may be arithmetically added and/or subtracted
  - Multiplication of ‘magnitudes’ and addition of ‘levels’ are equivalent, as are also subtraction of ‘levels’ and division of ‘magnitudes’
  - logarithmic units preserve relative errors (in contrast to linear units that preserve absolute errors)

dB reference

- Originally, dB was applied to signal power ratio (gain, attenuation)
- Then – to signal power level at a specified point and with respect to specified reference level:
  - Reference power levels
    - W → dB (or dBW)
    - mW → dBm (or dBmW)
- Now dB is applied also to other quantities, eg.:
  - Bandwidth: 10 MHz → 70 dBHz
  - Time: 2000 seconds → 33 dBs

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Various dBs

- **dBW: *dB*** referenced to one watt. *Zero dBW means one watt.*
- ***dBm: dB** referenced to one milliwatt. 'dBm' is often used in communication work as a measure of absolute power values. Zero dBm means one milliwatt.*
- ***dBμV : dB** referenced to 1 microvolt across a given impedance. Used often for receiver sensitivity measurement.*
- ***dBμV/m: dB** referenced to 1 microvolt per meter of electrical component of electromagnetic field of plane TEM wave. Used often for receiver sensitivity measurement.*
- ***dBi:** In the expression of antenna gain, the number of decibels of gain of an antenna referenced to the zero dB gain of a free-space isotropic radiator.*
- **Note**: There are also other 'dBs' in use!

Source: Telecommunication Glossary 2000

Simple equivalence

[W/W]	dB (W)	[W/W]	dB (W)
1	0	1	0
10	10	1/10	-10
100	20	1/100	-20
1000	30	1/1000	-30

Simple equivalence 2

%	dB
0.1	-30
1	-20
10	-10
100	0

Further equivalence

(with an error of ~1%)

dB (W)	[W/W]	dB (W)	[W/W]
0	1.0	6	4.0
1	1.25	7	5.0
2	$\pi/2$	8	2π
3	2.0	9	8.0
4	2.5	10	10.0
5	π	11	4π

Example: signal level

- A WLL access point power output is 20 dBm.
  - How many dBW is that?

$$P = 20 – 30 = -10 \text{ dBW}$$

(‘-30’ corresponds to 1/1000 or ‘milli’)

  - How many watts is that?

$$P = 10\text{^}(20/10) = 10^2 \text{ mW}$$
$$P = 10\text{^}(-10/10) = 10^{-1} \text{ W}$$

Example: noise floor

- Background noise power $P_N = kTB$

  k = 1.38 10^{-23} Joules/Kelvin (Boltzman's costant)

  T is absolute temperature in Kelvins

  B is the bandwidth in Hz

- At room temperature (290 K)

  kT = - 204.0 dBW/Hz

Example: noise floor (2)

- A radio link bandwidth is 20 MHz and noise temperature 290 K.
- What is the noise floor?

PN = kT [dB] + B [dB]

= -204 + (70 + 3)

= -131 dBW = -101 dBm

Example: sensitivity

- **Sensitivity** in radiocommunications is the minimum input signal required to produce a specified output signal following specified criteria (e.g. having a specified signal-to-noise ratio).
- It may be expressed as
  - Power in W, dBm or dBW
  - Field strength in micro- or milli-volts per meter or dB
  - Voltage across the input impedance in micro- or milli-volts or dB.

Example: sensitivity (2)

- Sensitivity of a radio receiver is -80 dBm
- How many watts it means?

  P = 10^(-80/10) mW

  $= 10^{-8} * 10^{-3}$ W = 10 pW (Prefix 'pico' = 10^{-12})
- Assuming input impedance of 50 ohms, how many volts it means?

  $U = \sqrt{(10^{-11} * 50)}$ V $= 10^{-6} * \sqrt{(5)}$ V = 2.236 μV

Example: SNR

- **Signal-to-noise ratio (SNR)** is the ratio of the desired signal to noise
  - usually expressed in dB
    - for impulse noise, expressed in terms of peak values
    - for random noise, expressed in terms of root-mean-square values.
  - When expressed in dB
    - is named also signal-to-noise-margin, or noise margin
    - is expressed as 20 times the logarithm of the amplitude ratio, or 10 times the logarithm of the power ratio.

Example: protection ratio

- **Protection ratio** is the minimum value of the wanted-to-unwanted signal(s) ratio at the receiver input, usually expressed in decibels
  - determined under specified conditions
    - such that a specified reception quality of the wanted signal is achieved at the receiver output
  - When expressed in dB
    - is named also protection margin
    - is expressed as 20 times the logarithm of the amplitude ratio, or 10 times the logarithm of the power ratio.

Example: PFD & Jansky

- PFD or Power Flux Density is radiated power passing through a given area expressed in W/m^2
- SPFD or Spectral Power Flux Density is PFD per unit bandwidth. Its units are [$W/m^2/Hz$] or Jansky.

1 Jansky = 10^{-26} $W/m^2/Hz$ → -260 $dBWm^{-2}Hz^{-1}$

Example: antenna aperture

- Aperture of isotropic antenna (unity gain in all directions) at wavelength λ:

  $\mathbf{A_i = (\lambda^2)/4\pi\ [m^2]}$

  (This is the area of a circle with a circumference of λ)

- Effective aperture of antenna with gain G_i:

  $\mathbf{A_e} = G_i * A_i\ [m^2]$

λ	A_i
1 m	-11 dBm^2
1 dm	-31 dBm^2
1 cm	-51 dBm^2
1 mm	-71 dBm^2

Note: A change of λ by 10x responds to change of A_i by 20 dB

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dB, voltages, currents

- Power ratios may be expressed in terms of
  - voltage and impedance, *E* and *Z*: $P = (E^2)/Z$
  - current and impedance, *I* and *Z*: $P = (I^2)*Z$
- Thus

$$X\ [\mathrm{dB}] = 10*\log_{10} \{(E_1)^2/Z_1)/(E_2)^2/Z_2)\}$$
$$= 10*\log_{10} \{(I_1)^{2*}Z_1)/(I_2)^2*Z_2)\}$$

dB, voltages, currents (2)

$$X\ [\text{dB}] = 20*\log_{10}(E_1/E_2) + 10*\log_{10}(Z_2/Z_1)$$
$$= 20*\log_{10}(I_1/I_2) + 10*\log_{10}(Z_1/Z_2)$$

- If – and only if -- $Z_1 = Z_2$, then

  $$X\ [\text{dB}] = 20*\log_{10}(E_1/E_2)$$
  $$= 20*\log_{10}(I_1/I_2)$$

Adding signals

- Let X1, X2, X3, … [dBW] be the individual signals power levels.
- The signals add to each other at the receiver input.
- What is the level of the resultant signal?
- Can we add dB directly?

Adding signals (2)

1. Translate each signal from [dBW] to [W]

$$x_i = 10\hat{}(\boldsymbol{X}_i/10) \text{ [W]}$$

2. Add signal powers in [W]

$$x_\Sigma = \sum x_i \text{ [W]}$$

3. Translate the result back to [dBW]

$$\boldsymbol{X}_\Sigma = 10 * \log_{10}(x_\Sigma)$$

dB vs. N

- Neper (N) is another logarithmic measure used in telecommunications
- Differences:
  - –Decibel relates to power ratio whereas Neper deals with voltage or current ratio
  - –Neper uses natural logarithms (base e) whereas deciBel uses decimal logarithms (base 10)

1N = 8,685890 dB

1dB = 0,115129 N

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Summary

- We learned
  - what is the logarithmic unit of dB and
- We saw few examples
  - how to use it

References

- Telecommunication Glossary 2000
- Davis M: “Units and Calculations”; presentation at the Summer School on Spectrum Management and Radio Astronomy, Green Bank, June 2002