# A TV White Space Broadband Market Model for Rural Entrepreneurs

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*Abstract*—**The paper presents an analytic TV white space market model for rural entrepreneurs for provisioning broadband internet access within the context of a low cost design of a smart mesh network. The smart mesh is a hybrid network of WiMAX base station (BS) and WiFi Access points (APs) that belong to different entrepreneurs participating in TV white space trading. This interaction of entities is viewed as a pricing problem where a BS seeks to maximize profit by selling secondary spectrum and WiFi APs strive to satisfy their customers demand by acquiring as much secondary spectrum as possible. The pricing problem is subsequently formulated as a leader-follower Stackelberg game in an oligopolistic market, with WiMAX BS station as leader and APs as followers. Inter-operator agreement between Leader and follower is based on throughput as the Quality of service (QoS) metric. Analytically a direct dependence between the throughput and price paid exists, meaning users are able to determine their QoS based on willingness to pay. The implication of the model is that of its applicability in spectrum management and provision of broadband Internet access to schools in rural and remote areas like the Limpopo province in South Africa.**

## I. INTRODUCTION

Wireless mesh networks (WMNs) are envisioned as a candidate technology that is set to facilitate ubiquitous connectivity to the end user, especially in the isolated areas of the developing world and rural areas of the developing countries where cash strapped Internet service providers (ISPs), carriers, and other operators may find incentives for using this technology for rolling out robust and reliable wireless broadband service access in a way that needs minimal up-front investments. Building upon a mesh structure that comprises nodes in the form of mesh routers and mesh clients, they have the ability to dynamically self organize as well as self configure, with the nodes in the network automatically establishing and maintaining mesh connectivity among themselves. This feature combined with their low upfront cost, easy network maintenance, robustness, and reliable service coverage [1] have led to their adoption as in numerous deployments such as broadband home networking, community and neighbourhood networks, enterprise networking, building automation etc. However, current WMNs are typically characterised by a static spectrum allocation policy defined by government agencies where wireless spectrum is assigned to license holders on a long time basis for large geographical regions [2]. This has constrained the operation of these networks to a narrow spectrum band where they are subjected to competition for spectrum availability with the data hungry smartphones and new machine to machine applications [3]. Specifically this development has brought about the fundamental problem of spectrum scarcity. Pursuant to this problem, empirical spectrum occupancy measurement efforts have revealed an inefficient utilization of spectrum evidenced by the existence of large portions of spectrum with little usage within a wide range of frequency bands [5]. When these bands are dynamically viewed as a function of time (temporal variations) and space (spatial variations) or both, there exists spectrum opportunities or holes that require more efficient techniques for full exploitation [6]. Moreover, some of the licensed bands such as those allocated to TV broadcasting which are usually most protected through regulation have revealed an inefficient frequency usage contrasting with the over-crowding and over-usage of other bands such as Industrial Scientific and Medical (ISM) band. This discrepancy between usage and allocation has raised the need for a redesign of frequency regulations by leveraging on the recent technological innovations such as Smart radio [7] to enable dynamic spectrum access (DSA). DSA has been touted as a viable candidate solution to the spectrum scarcity problem. Clearly, DSA presents new unique opportunity of spectrum sharing among operators or dissimilar technology systems as it renders feasible inter-operator agreements based on Quality of Service(QoS) metrics for improved revenue and efficient spectrum utilisation. The implication, is that of a joint spectrum management process in which dynamic and coordinated management of spectrum resource for different radio access technologies(RATs) is enabled. Such a strategy may be activated within single or multiple operator networks in order to support multiple objectives, e.g avoiding disconnections due to lack of coverage in the current RAT, blocking due to the overload in the current RAT, possible improvement of QoS by changing the RAT and support of users and operators preferences [8]. Coincidentally for less developed regions, [9] reports two main areas of policy concern to the African region as:

- To sustain mobile cellular and internet growth use as well as extend access to lower income segments of the

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**Acknowledgement:** This paper was supported by the project "Secured quality of service aware wireless sensor networks: toward effective network and energy management", funded by a joint-project grant from the South African NRF and Algerian DGRSDT.

population.
- To take the necessary steps to enable greater broadband access.

The same authors further recommend the challenges to be faced to achieve in order to promote infrastructure sharing, lower costs, promote wireless broadband and expand public internet access. Clearly, increased levels of Information and Communications technology(ICT) access are easy to promote through public facilities such as community access centres, internet cafes and schools. To this end, such an opportunity has been identified in Cape Town, South Africa [10] where at the moment of the writing of this paper, the first trials are underway to assess the relevance of using RAT technologies and take advantage of TV white spaces to improve Internet connectivity in the educational sector. Elsewhere in the Limpopo province Microsoft's pilot project is aimed at building a TV white space based network. The TV white space based broadband network comprises solar powered base stations to deliver Internet to five secondary schools in rural and remote parts of the Limpopo province [4]. Each of the five schools is to be equipped with Windows tablets, projectors, teacher laptops and training, education-related content, solar panels for device charging where there is no access to electricity, and other support .

As an extension to the work presented by [10], this paper presents a market model to be used by rural entrepreneurs when providing Internet access to schools in the rural areas. We present an analytic model for the design and performance evaluation of a low cost smart WMN for broadband internet access. We consider a game theoretic view of an integrated multi-operators WiMAX/WiFi network where a WiMAX base station is used to provide secondary spectrum access to a number of WiFi routers in a network interaction generalised as a Stackelberg leader-follower game where the WiMAX base station(BS) takes the form of a leader while the WiFi access points are followers. Our work uses a game theoretical model which is similar to [18]. However, in contrast to [18] where the delay is used as QoS metric to provide real-time data such as voice communication, our point of departure is that of an inter-operator agreement based on QoS throughput metric with focus on providing data to improve the downloading of information on the Internet which is a major focus in research and educational institutions. The implication of the model is that of its applicability in spectrum management and provision of broadband internet access to schools in cities and rural areas.

The rest of the paper is organised as follows. Section 2 presents the TV white space model including an inter-operator spectrum sharing framework for multiple operators/Radio Access Technologies (RATs) and the game theoretical approach. The market pricing model is formulated in section 3 while the analytical results and conclusion are presented in sections 4 and 5 respectively.

## II. THE TV WHITE SPACE MARKET MODEL

Some microwave links use the frequency bands that have been allocated for unlicensed use, in the ISM band, which stands for Industrial, Scientic and Medical. Most other parts of the electromagnetic spectrum are tightly controlled by the licensing legislation, with license cost being a huge economic factor. In most countries, the ISM bands can be used without paying license fees. The WiFi system uses ISM bands (2.4 GHz and 5.8 GHz) while the WiMAX system works over a wide variety of frequencies, mostly in licensed bands. Some vendors have developed WiMAX equipment that use the 5.4 and 5.8 GHz license-exempt ISM bands while having the advantages of the WiMAX ecosystem, which includes Quality of Service (QoS) assurance and a media access control (MAC) better suited for long distance communication. Such equipment offers real-time outage-free video, voice and data transmission services, while implementing handover mechanisms and 100 percent QoS guarantee, with up to 50 Mbps gross capacity. They thus provide the potential to be used as backhaul for WiFi mesh networks without requiring the use of licensed bands.

### *A. Inter-Operator Spectrum Sharing Framework*

In order to achieve spectrum sharing between multiple operators/Radio Access technologies (RATs), the authors in [11] use an integrated WiMAX/WiFi architecture. The architecture comprises of WiMAX Base Station (BS), Subscriber Stations (SSs) and WiFi access points (APs)/router. The BS operates in a licensed band servicing both WiMAX SSs and WiFi APs. The connection between BS and SS is dedicated to a single user, with the connection between AP/router and BS being apportioned among the wireless LAN(WLAN) nodes. For concreteness, the WiMAX and WiFi APs/routers are operated by different service providers. The challenges to be encountered in the quest for a perfect integrated WiMAX and WiFi networks are perceived as technical and economic. The technical component encompasses the issues of protocol adaptation and QoS. The economic component pertains to the control of radio usage which subsequently translates to a pricing issue. Thus [12] describes pricing in spectrum trading as a key issue of interest to primary service providers as well as to secondary service providers. In literature, the pricing problem has been approached from two perspectives, namely that of optimization formulation and game theory. With regards to the optimization based pricing, [13] cites the difficulty in designing suitable distributed algorithms that are capable of ensuring stability as well as convergence to a global optimum as the main drawback of this approach. Game theory as an alternative approach has been employed in the analysis of resource management in telecommunications networks for at least 20 years. As game theory has been traditionally applied to economic problems, it is not surprising that one of its first applications in telecommunications was in the study of pricing [14]. [15] concurs by describing pricing as a commonly used incentive technique in game theory. The effectiveness of this technique, the authors argue, depends on the selection of the pricing function, which is itself a challenging problem.

### B. *Game Theoretic Approach*

Unlike optimization, the game theoretic formulation strives to attain individually optimal solutions, in a scenario where multiple entities are concerned. The major components of a game include, players, strategies and payoff of the players concerned with the game. In such games various players choose their own strategies with ther payoff of the players denoting the game outcome. Objectively a game is aimed at obtaining a strategy profile such that all concerned players are satisfied.Such a scenario is perceived as Nash Equilibrium. Formally Nash Equilibrium is defined a collection of the set of strategies for all concerned players. Other notable solution concepts in this category include the max-min solution and Stackelberg equilibrium, the former solution guarantees maximisation of minimum payoff while the latter ensures maximisation of the leader's pay. Recently, game theoretic approach has become a useful and powerful tool in research on wireless mesh networks. Game theory is the branch of decision theory concerned with interdependent decisions. The problems of interest involve multiple entities, each of whom has individual objectives related to a common system or shared resources. Because game theory arose from the analysis of competitive scenarios, the problems are called games and the participants are called players. A suitable example in this context is that of a wireless network comprising service providers intent on maximising their profits while users strive to achieve their off payoff.

Recently, numerous works on the game theoretic approach to the pricing have been recorded [16] and [17]. In the former two scenarios are studied in a WiFi network, the scenarios are that of a direct interaction between AP and client in the first instance and later interaction through a reseller. In the latter work,pricing is studied in a mesh network setup in which mesh nodes relay traffic to a gateway. Intermediate routers are modelled as bandwidth sellers, when selling bandwidth to client nodes which is obtained from a gateway. Two case scenarios are studied, firstly when the system model comprises unlimited link capacity and secondly when there is limited link capacity. The authors did not ignore the issue of a variable number of connections in adjacent routers and clients. Ultimately an optimal price was however arrived at, on the basis of ongoing connections. Other notable efforts have however progressed towards an integrated WiMAX/WiFi network, given the hype about 3G networks and WiMAX's candidancy as a solution for broadband access. A case in point is in [18] in which such a problem of radio spectrum management/sharing is addressed in such a setup. The hierarchical spectrum allocation based on game theory was formulated. The problem is further decomposed into sub games, with each game being solved at the appropriate level. These efforts have been extended in for a scenario which an integrated WiMAX/WiFi is considered, with the WiMAX being used as a backhaul for WiFi spots. These efforts regrettably, ignore the pricing issue and the authors in [11] take it upon themselves to tackle the pricing issue for such an integrated network. The scope of these efforts is limited in that only the delay is used as the Qos metric. Certainly other metrics should be considered in order to validate such a model.

TABLE I
REVENUE AND ELASTIC DEMAND

| **Symbols** | **Description** |
|---|---|
| $b_i$ | bandwidth |
| $P_M^{(r)}$ | Price |
| $c_i$ | constant(fixed bandwidth) |
| $d_i$ | constant(elasticity) |
| $a_i$ | constant(fixed revenue) |
| $e_i$ | constant(rate of revenue change) |
| $R_M^{(r)}$ | Revenue(WiFi) |
| $C_M^{(r)}$ | Cost |
| $P_M^{(BS)}$ | Price charged by WiMAX BS |
| $F_M^{(r)}$ | Fixed Cost for WiFi router |
| $T$ | Throughput |
| $\phi_M^{(r)}$ | Profit(WiFi) |
| $\phi^{(BS)}$ | Profit(WiMAX BS) |
| $n$ | number of users |
| $\beta$ | constant |

## III. THE MARKET PRICING MODEL

An envisaged model for spectrum sharing in which dynamic spectrum access is enabled due to an inter-operator agreement between WiMAX BS and WiFi networks based on QoS metric for improved revenue and efficient spectrum utilisation is proposed. The arrangement is that of a WiMAX BS being a primary user and charging WiFi AP/routers for the shared use of its licensed spectrum when providing clients with broadband internet as shown in Figure 1. Unlike the SSs which subscribe at a flat rate due to their fixed bandwidth requirement, the WiFi nodes have an elastic demand, which is a function of the number of nodes as well as preference. The WiMAX thus charges the WiFi nodes with variable pricing (e.g. P1 and P2 respectively). The pricing problem is subsequently generalised into a Stackelberg game in which the WiMAX profit is maximized and the WiFi routers are satisfied with the bandwidth sharing and pricing. Stackelebrg equilibrium is the solution to this game and is easily obtainable when all information about the service providers and clients is available.

However in reality the WiMAX BS may not be familiar with the preferences of the WiFi AP/routers. Similarly the WiFi router may also not be familiar with the bandwidth demand functions of its nodes and may thus need to learn from historical data to achieve equilibrium. We adopt a Darwinian approach as in which a genetic algorithm is implored for learning to achieve the desired objective. To understand the notation used in the numerous functions, that we are going to use, a number of symbols as well as their meanings in are described in accordance with Table I.

The WiMAX BS's utility translates to the revenue as well as the cost of the spectrum in terms of price. To this end, revenue is deemed proportional to the satisfaction of the users(QoS), while cost of the spectrum is guided by the law of supply and demand. Specifically, the WiMAX BS charges different prices to different WiFi APs/routers depending on the bandwidth demand from WiFi clients. With regards to SSs, throughput

[19] is the implored QoS metric, such that the WiMAX BS's revenue is expressed as:

$$R^{(s)} = \sum_{i=1}^{N} [a_i - e_i T(n, b_i^{(s)})]$$

The throughput is expressed as:

$$T(n, b_i^{(s)}) = \sum_{i=1}^{N} \frac{\beta b_i^{(s)}}{\sqrt{nlogn}}$$

The price charged by the WiFi AP/router tends to influence the level of demand by a WiFi node. The demand function is linear and is expressed as follows:

$$b(P_M^{(r)}) = c_i - d_j P_M^{(r)}$$

The fixed bandwidth ci, is directly proportional to the bandwidth demand, if it is high , the bandwidth demand is correspondingly high. As for the WiFi network, its revenue is obtainable from

$$R_M^{(r)} = \sum_{j=1}^{N_M^{(r)}} P_M^{(r)} b_j(P_M^{(r)})$$

with the cost calculated from

$$C_M^{(r)} = P_M^{(BS)} \sum b(P_M^{(r)}) + F_M^{(r)}$$

### Stackelberg Game and Profit Maximization

A Stackelberg game which is also known as a leader-follower game, was originally used by Stackelberg in 1952 based on some economic monopolistic market phenomena. In such a game modelling the market, decision making is sequential with a player , called the leader committing a strategy followed by other players called followers. The equilibrium of this formulation is obtainable through backward induction. The leader in this kind of game considers the best response of the follower. On the basis of the follower's decision, the leader then chooses the optimal supply quantity that will yield higher returns(profit). When adapted for the spectrum sharing scenario, the objective is to achieve an equilibrium on spectrum sharing and pricing between the WiMAX and WiFi service providers. To this end, based on the assumptions that WiMAX and WiFi are rational to maximize their profits, the game is formulated as follows:

**Players:** The WiMAX BS(Leader) and WiFI AP/router(Followers) as depicted in Figure 1

**Strategies:** The WiMAX BS strategizes through the price, P(BS)M , charged to the WiFi APs, while the WiFi AP's strategy is the required bandwidth.

$$b_M^{(r)} = \sum_{i=1}^{N} b_i(P_M^{(r)}) \tag{1}$$

**Payoffs:** For both players, the payoffs are the corresponding profits. For a credible action, we first familiarize with the payoff for the WiFi AP/router. On the basis of the price charged by the WiMAX BS, the profit of router is gibe by

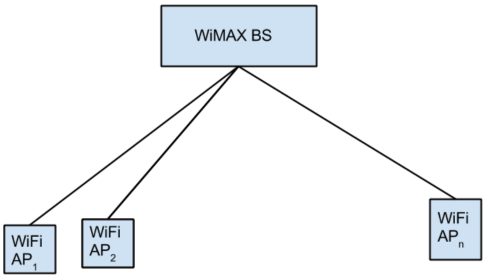


Fig. 1. Leader-Follower Stackelberg

$$\begin{aligned}\phi_M^{(r)} &= R_M^{(r)} - C_M^{(r)} \\ &= \sum_{i=1}^{N} P_M^{(r)}(c_i - d_i P_M^{(r)}) \\ &- P_M^{(BS)} \sum N_{i=1}(c_i - d_i P_M^{(r)}) - F_M^{(r)}\end{aligned}$$

Using simple calculus, the optimal price charged to a WiFi node is obtainable by way of computing the derivative of the profit function and then setting to zero. Given the price $P^(k)M$, the bandwidth function then computes bandwidth demand for all nodes within the vicinity of hotspot M.On the basis of the perceived response of the WiFi AP/router, the WiMAX BS gains the leverage to adjust its price $P^{(r)}M$, charged to router M so as to achieve the highest possible payoff. The WiMAX BS payoff is defined as:

$$\begin{aligned}\phi^{(BS)} &= R^{(s)} + \sum N_{M=1} R_M^{(r)} \\ &= \sum N_{i=1}[a_i - e_i T(n, b_i^{(s)})] \\ &+ \sum N_{M=1} P_M^{(BS)} b_M^{(r)}\end{aligned}$$

The Stackelberg equilibrium is a solution concept originally defined for the scenarios where a hierarchy of actions exists between users. To this end, the Stackelberg contextually prescribes an optimal strategy for the leader, so long as its followers always respond by playing their nash equilibrium strategies in the smaller subgame. Nash equilibrium is thus considered as the solution of the bandwidth sharing and pricing game which guarantees the maximization of the WiMAX BS. However in a scenario, where all information is completely known, calculus is used to obtain the derivative of the WiMAX BS and then computing the price, this is regrettably not so for a practical system and other avenues should be pursued to learn user's preferences. This can only be achieved through the use of a Darwinian approach.

### Darwinian Approach

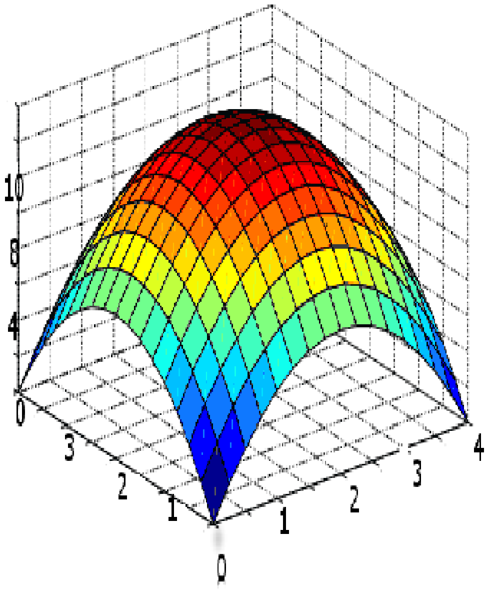
The Darwinian approach involves the use of a genetic Algorithm(GA), which are founded on the principles of natural evolution and selection. In the general case, an initial population is generated, followed by computing and saving the fitness of each chromosomes in the current population. With the aid of an appropriate selection mechanism, chromosomes are picked from a pool to produce an offspring via crossover and mutation operators. The process is continued until such a time that a satisfactory solution is obtained or alternatively a maximum generation number is reached. Within the context of the Stackelberg game, an interaction between the WiMAX BS and WiFi router/AP enables the GA to obtain information about bandwidth demand and adjust prices appropriately. Our Fitness function is the profit function for the WiMAX (BS) and the chromosomes are the prices charged by the BS i.e P1 and P2. The information interaction between WiFi router/AP and its nodes, triggers a response from each WiFi node with regards to bandwidth demand. The process continues until WiFi AP/router profit is maximized. Conversely the WiMAX BS broadcasts information about the price charged to achieve the same.

## IV. Results

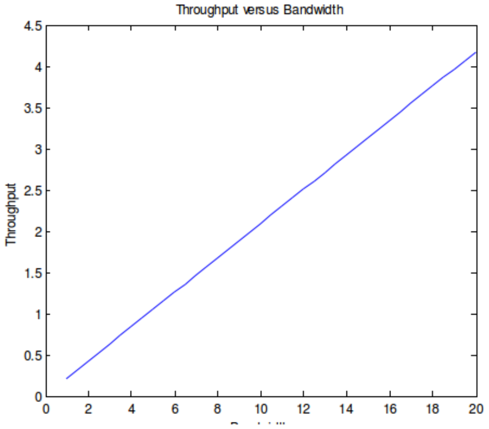
In this section we present and discuss our results obtained through numerical performance analysis of the model. The analysis is as shown in Fig 2 through Figure 4. In Figure 2(a), the profit function which is price dependent takes the form of a reverse De jong form. The point at which this profit function is maximal is the equilibrium form. However, from a Genetic algorithm perspective, a more practical approach yields an equilibrium value for the BS profit in approximately fifty generations in Figure 3. This value as expected is lower than the theoretically predicted one in the reverse De Jong function. The throughput is our QoS metric in this analysis and it thus suffices to give this metric more attention. In Figure 2(b), the throughput varies directly with the bandwidth, i.e as more bandwidth is availed to an AP/router, the network is able to increase its throughput. However if bandwidth is held stagnant at some point and the number of users varied, the throughput is initially notably high and subsequently falls almost exponential with an increase in the number of users as shown in Figure 4(a). Clearly is this to be expected as less bandwidth is availed, this is predicted by the previous graph. The ideal scenario step to take is to purchase more bandwidth to increase throughput and for this reason Fig 4(b) (throughput-price) depicts the variation of throughput with price. From Fig 4(b), we note that the throughput increases linearly with an increase in price of bandwidth. More has to be paid for an increased output. This implies the QoS is price dependent as predicted earlier, a subscriber will determine their QoS by their willingness to pay. There is however an exception in that at a price value of 5, the throughput is nearly the same irrespective of the amount of bandwidth.

## V. Conclusion

In this paper we have presented an analytic TV White Space broadband market model for use by rural entrepreneurs in provisioning broadband internet access to rural and remote areas. The hybrid smart Mesh has been designed based on a generalised Stackelberg game. The design facilitates an efficient sharing between the players, namely the WiMAX BS and WiFi Access points/routers. The spectrum sharing is based on a pricing scheme, in which the throughput is used as QoS metric. The throughput varies directly with price so much that, users can determine how much throughput they require based on what they are willing to pay. The model finds applications in the provision of broadband internet services for public entities like schools in rural and remote areas with the Limpopo province in South Africa being a case in point. Further investigation efforts need be focussed on extending the model to a scenario of multiple leaders and followers. The work presented in this paper can also be used as a foundation to either extend market-pricing schemes such as presented in [20] in hybrid wireless-optical networks where optical broadband islands are interconnected by a heterogeneous WiMAX-WiFI wireless infrastructure. It can also serve as a basis for pricing bandwidth in traffic engineering schemes such as previously proposed in [21]–[24].



(a) Profit



(b) Throughput

Fig. 2. *Profit and Throughput*

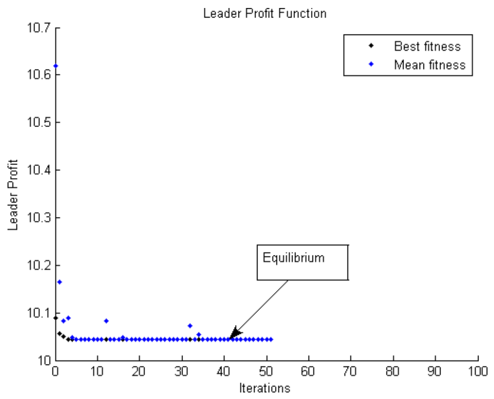
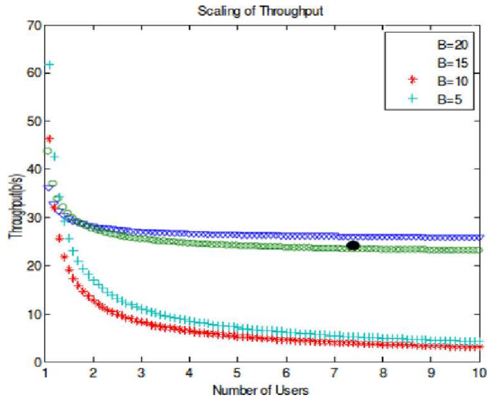
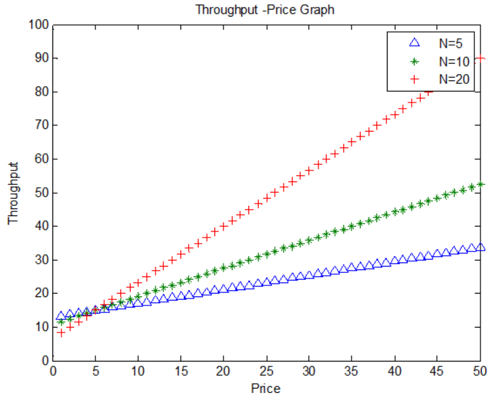


Fig. 3. Profit



(a) Throughput Scaling



(b) Throughput Pricing

Fig. 4. *Throughput Scaling and Pricing*

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